Enabling Puncturable Encrypted Search over Lattice for Privacy-Preserving in Mobile Cloud

Yibo Cao, Shiyuan Xu, Gang Xu, Xiu-Bo Chen, Zongpeng Li, Jiawen Kang, and Dusit Niyato

Abstract—Searchable encryption (SE) has been widely studied for mobile cloud computing, allowing data encrypted search. However, existing SE schemes cannot support the fine-grained searchability revocation. Puncturable encryption (PE) can revoke the decryption ability for a specific message, which can potentially alleviate this issue. Moreover, the threat of quantum computing remains an important concern, leading to privacy leakage in the mobile cloud. Consequently, designing a post-quantum puncturable encrypted search scheme is still far-reaching. In this paper, we propose **PunSearch**, the first puncturable encrypted search scheme over lattice for data privacy-preserving in mobile cloud. PunSearch provides a fine-grained searchability revocation while enjoying quantum safety. Different from existing PE schemes, we construct a novel trapdoor generation mechanism through evaluation algorithms and pre-image sampling technique. We then design a search permission verification method to revoke the searchability for specific keywords. Furthermore, we formulate a new IND-Pun-CKA model and utilize it to analyze the security of PunSearch. Comprehensive performance evaluation indicates that the computational overheads of Encrypt, Trapdoor, Search, and Puncture algorithms in PunSearch are just 0.064, 0.005, 0.050, and 0.311 times of other prior arts, respectively under the best cases. These results demonstrate that PunSearch is effective and secure in mobile cloud computing.

Index Terms—Mobile cloud computing, puncturable encrypted search, data privacy-preserving, lattice.

1 Introduction

Mobile cloud computing is a promising solution that provides data storage and access services over mobile networks, which has become an indispensable component of current data management (e.g., Google Drive, Aliyun) [1], [2], [3]. Specifically, data senders have the ability to outsource their data to the cloud through various mobile devices, thereby reducing local maintenance costs [4]. In addition, data receivers can search data from mobile cloud servers, facilitating convenient data sharing and retrieval [5]. Meanwhile, mobile cloud servers are usually semi-honest, causing data privacy leakage [6], [7], [8]. Although the encryption-before-outsourcing technique can protect data privacy to some extent [9], [10], [11], it heavily limits data availability, which can be detrimental to data search and sharing [12]. To ensure data privacy without losing its usability, a cryptographic primitive named searchable encryption (SE) was formalized. In Fig. 1, through the SE technique [13], [14], [15], data receivers can obtain the search result and apply it to a variety of mobile scenarios (e.g., mobile healthcare, mobile transportation, mobile office). In recent years, numerous researchers have introduced SE in cloud, thereby enhancing the functionality and security of SE, e.g., conjunctive keyword SE [16], multi-receiver SE [17], [18], authenticated SE [19], [20], etc.

Nevertheless, the aforementioned schemes only provide immutable searchability for data receivers, which renders them unsuitable for mobile scenarios. To address this problem, an intuitive approach is to revoke the searchability promptly. For example, multiple departments affiliated within an enterprise often outsource numerous private datasets to the mobile cloud, and the data accessibility to each department is subject to change over time. When a department is no longer engaged in these activities, the enterprise could revoke its searchability to limit its data access permission, which is essential for mobile cloud systems. Furthermore, the advent of quantum computing presents significant obstacles to data privacy-preserving in mobile cloud [21], [22]. In particular, quantum computers can break the SE schemes based on the classical cryptographic assumption (e.g., discrete logarithm) [23]. Consequently, a substantial amount of encrypted private data will be disclosed, resulting in serious breaches of data confidentiality.

There still exist two challenges to be addressed in encrypted search for mobile cloud computing. The first challenge is how to construct an SE scheme with searchability revocation. A straightforward approach is to introduce the receiver revocation [24], [25] and forward security [26], [27], [28], [29] into SE, which is designed to revoke a data receiver's searchability. However, this mechanism is all-or-nothing and cannot realize fine-grained searchability revocation for specific keywords. To bridge the gap,

- *Y. Cao and X.-B. Chen are with the Information Security Center, State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China. (E-mail: caoyibo@bupt.edu.cn, flyover100@163.com).*
- *Y. Cao and S. Xu are with the Department of Computer Science, School of Computing and Data Science, The University of Hong Kong, Pok Fu Lam, Hong Kong 999077. (E-mail: syxu2@cs.hku.hk).*
- *G. Xu is with the School of Artificial Intelligence and Computer Science, North China University of Technology, Beijing 100144, China, and also with the Information Security Center, State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China. (E-mail: gx@ncut.edu.cn).*
- *Z. Li is with the Institute for Network Sciences and Cyberspace, Tsinghua University, Beijing 100084, China. (E-mail: zongpeng@tsinghua.edu.cn).*
- *J. Kang is with the School of Automation, Guangdong University of Technology, Guangdong 510006, China (E-mail: kavinkang@gdut.edu.cn).*
- *D. Niyato is with the College of Computing and Data Science, Nanyang Technological University, Singapore 639798. (E-mail: dniyato@ntu.edu.sg).*

puncturable encryption (PE) provides the revocation of decryption ability for a specific message, which motivates this work to design a puncturable SE scheme to realize the encrypted search. Subsequently, the second challenge is how to ensure this puncturable encrypted search scheme that can withstand quantum computing attacks. To address this concern, lattice-based cryptography has been widely adopted. Many researchers have designed various lattice-based SE primitives [30], [31], [32], and used them for data privacy-preserving. To the best of our knowledge, there is no post-quantum puncturable SE scheme, which has become another motivation for us. These motivations guide us to ask a question about this work:

Can we propose a post-quantum puncturable encrypted search scheme for mobile cloud data privacy-preserving?

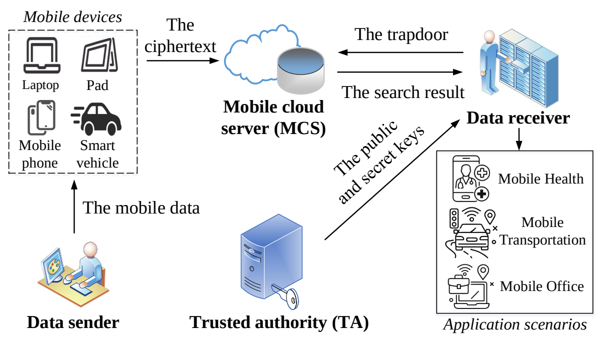


Fig. 1. The mobile cloud data encrypted search procedure. The data sender encrypts and stores its data on a mobile cloud server via a mobile device, and the data receiver submits a search trapdoor to realize the encrypted search, and then applies the search results to various mobile scenarios.

In this paper, we address the aforementioned challenges with novel solutions. We propose **PunSearch**, the first lattice-based puncturable encrypted search scheme for outsourced data privacy-preserving in mobile cloud. PunSearch achieves the fine-grained searchability revocation for specific keywords while resisting quantum computing attacks. For puncturing the secret key, we introduce lattice basis generation algorithms ExtendLeft and RandBasis to obtain a lattice basis as a punctured secret key. Based on this, our main challenge is how to generate a search trapdoor from a punctured secret key and realize the encrypted search. Existing lattice-based SE schemes have utilized the SampleLeft or SamplePre algorithms to generate the trapdoor [26], [30], [31], [22], but this punctured secret key cannot be used as input for these algorithms. Thus, addressing this challenge is not trivial. Different from the existing schemes, we leverage the evaluation algorithm Eval_{pk} with the GenSamplePre technique to design a novel trapdoor generation mechanism. Specifically, we first invoke the Eval_{pk} algorithm to map a punctured tag list P into several matrices. Then, we utilize these matrices and the punctured secret key as the input of GenSamplePre technique to obtain a trapdoor. For the search phase, we need to verify the search permission to determine whether this trapdoor has the searchability for a specific ciphertext. If so, we execute the ciphertext search and return a result to the data receiver.

We summarize our contributions as follows.

- We first present a puncturable encrypted search scheme over lattice for mobile cloud, named PunSearch. Our scheme not only supports the fine-grained revocation of searchability for specific keywords, but also resists quantum computing attacks, which adapts for data privacy-preserving in mobile cloud computing.
- We design a novel architecture for the searchable encryption scheme in the mobile cloud by introducing a secret key puncture procedure and constructing a designated trapdoor generation mechanism. Specifically, we first introduce the lattice basis generation algorithm to revoke the searchability of a data receiver for specific keywords, and then utilize the evaluation algorithm and lattice pre-image sampling technique to generate a search trapdoor. Moreover, we devise a permission verification method to determine the searchability of this trapdoor.
- We also formalize the security notation of PunSearch for the first time. Then, we give a rigorous security analysis to demonstrate that PunSearch provides IND-Pun-CKA security in the random oracle model (ROM), which can be reduced to the Learning With Errors (LWE) assumption, thus the security of mobile cloud data can be ensured.
- Comprehensive performance evaluation indicates that PunSearch is more efficient than existing methods [26], [30], [31], [22], [33], [34] in the context of computational overhead. In particular, the time cost of **Encrypt**, **Trapdoor**, and **Search** algorithms in PunSearch are just 0.064, 0.005, and 0.050 times compared to other lattice-based SE schemes [26], [30], [31], [22], respectively for the best cases. Besides, the time cost of our **Puncture** algorithm is only 0.311 times of other lattice-based PE schemes [33], [34], which is practical for mobile cloud computing.

The remainder is structured as follows. Section II summarizes the literature review, and Section III presents the basic concepts. In Section IV, we present the problem formulation. The concrete design of our PunSearch scheme is elaborated in Section V. Sections VI and VII cover the security analysis and performance evaluation, respectively. Eventually, we conclude this paper in Section VIII.

2 RELATED WORKS

2.1 Searchable Encryption

Searchable encryption (SE) technique supports encrypted search for mobile cloud computing, which has garnered widespread attention. Boneh et al. [35] formalized the SE scheme in a public key setting, named public key encryption with keyword search (PEKS). Since then, various SE schemes have been proposed for cloud data privacy-preserving. Xu et al. [36] presented an authorized SE primitive to protect the privacy of user identity and encrypted data. For cloud e-mail servers, a more practical multi-keyword SE scheme with hidden structures (PMSEHS) was designed by Xu et al. [37] to achieve the encrypted e-mail search as fast as possible. Zhang et al. [38] presented a

TABLE 1
Functionality comparison with the existing state-of-art schemes

Schemes	Encrypted Search	Searchability Revocation	Secret Key Puncture	Quantum Resistance
Boneh et al. [35]	✓	✗	✗	✗
Xu et al. [36]	✓	✗	✗	✗
Xu et al. [37]	✓	✗	✗	✗
Zhang et al. [38]	✓	✗	✗	✗
Sun et al. [24]	✓	✓	✗	✗
Zhang et al. [25]	✓	✓	✗	✗
Lu et al. [27]	✓	✓	✗	✗
Zhang et al. [26]	✓	✓	✗	✓
Zhang et al. [21]	✓	✗	✗	✓
Luo et al. [30]	✓	✗	✗	✓
Luo et al. [31]	✓	✗	✗	✓
Lin et al. [22]	✓	✗	✗	✓
Green et al. [39]	✗	✗	✓	✗
Phuong et al. [40]	✗	✗	✓	✗
Ghopur et al. [41]	✗	✗	✓	✗
Cui et al. [42]	✗	✗	✓	✗
Susilo et al. [43]	✗	✗	✓	✓
Dutta et al. [33]	✗	✗	✓	✓
Dutta et al. [34]	✗	✗	✓	✓
Yang et al. [44]	✗	✗	✓	✓
Yang et al. [45]	✗	✗	✓	✓
Our PunSearch	✓	✓	✓	✓

Note: ✓means this scheme enjoys this functionality; ✗means this scheme does not enjoy this functionality; Secret Key Puncture means this scheme can realize search or decrypt ability revocation for a specific keyword or message, more fine-grained than other revocation schemes.

subversion-resistant and consistent attribute-based SE system, which is designed to resist several external attacks. Nevertheless, these schemes cannot support the searchability revocation of the data receiver.

To alleviate this, many researchers proposed receiver revocation SE [24], [25]. Sun et al. [24] utilized proxy-based encryption to put forward an attribute-based SE scheme supporting user revocation. Zhang et al. [25] designed a novel secure search method under a multi-owner model, which provides ranked multi-keyword search and efficient receiver revocation. Alternatively, Lu et al. [27] introduced the forward security to construct a forward authenticated SE scheme with a fieldless concatenated keyword, which can resist keyword guessing attacks.

Moreover, lattice-based SE primitives were proposed to cope with quantum computers [26], [21], [30], [31], [22], [46]. Concretely, Zhang et al. [26] presented a forward secure SE method over lattice, namely FS-PEKS, applied in the cloud-assisted industrial Internet of Things (IIoT). They [21] then devised a biometric identity-based SE scheme supporting multi-keyword search (BIB-MKS) over lattice. Following this, Luo et al. designed a lattice-based attribute-based authenticated SE (ABAEKS) [30] and proxy-based authenticated SE (Re-PAEKS) [31] schemes, which are resistant to internal keyword guessing attacks. For the multi-keyword search scenarios, Lin et al. [22] constructed three lattice-based SE schemes that support flexible keyword search. However, the above-mentioned solutions forgot to consider the fine-grained searchability revocation for specific keywords, which limits the practicality in the mobile cloud.

2.2 Puncturable Encryption

Puncturable encryption (PE), formalized by Green et al. [39], can revoke the decryption ability for a specific message. Phuong et al. [40] combined PE and attribute-based encryption (ABE) scheme to propose a new punctured keys generation method. To enable the encrypted data with decryption ability revocation, many PE schemes were introduced in cloud [41], [42]. For instance, a revocable ABE scheme incorporated PE to realize the receiver revocation [41]. Meanwhile, Cui et al. [42] presented a PE primitive to support the secret key self-update, enhancing its practicability.

However, the above-mentioned schemes forgot to consider quantum computing attacks. To address this, many researchers have constructed several PE schemes based on lattice hardness [33], [34], [43], [44], [45]. Susilo et al. [43] pointed out that the puncture property can be constructed through efficiently computable functions, and presented the first lattice-based PE scheme. Following this direction, Dutta et al. [33] offered puncturable identity-based encryption (PIBE) over lattice, and extended it to the puncturable key-policy ABE scheme. Subsequently, Dutta et al. also [34] put forward a lattice-based hierarchical PIBE scheme that supports more general key updates and flexible secret key puncture. For cloud systems, to avoid unauthorized access, Yang et al. designed an innovative lattice-based puncturable ciphertext-policy attribute-based encryption (CP-PABE) scheme [44] and a puncturable attribute-based matchmaking encryption (PM-ABE) [45] scheme, achieving data privacy-preserving in the post-quantum era.

Unfortunately, as shown in Tab. 1, none of the existing PE schemes provides the search property. Thus, it is necessary to design a puncturable encrypted search scheme for mobile cloud while resisting quantum computing attacks.

3 Preliminaries

Definition 1. We define the basis of lattice Λ is a matrix $\mathbf{A} = (\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_m) \in \mathbb{R}^{n \times m}$ with linearly independent columns vectors, s.t. $\Lambda = \Lambda(\mathbf{A}) = \{x_1 \cdot \mathbf{a}_1 + x_2 \cdot \mathbf{a}_2 + \ldots + x_m \cdot \mathbf{a}_m | x_i \in \mathbb{Z}\}$.

Definition 2. Given three integers $n, m, q \in \mathbb{Z}$ and a matrix $\mathbf{M} \in \mathbb{Z}_q^{n \times m}$, we give the definition of the q-ary lattice: $\Lambda_q^{\perp}(\mathbf{M}) := \{\mathbf{r} \in \mathbb{Z}^m | \mathbf{M}\mathbf{r} = \mathbf{0} \bmod q\}$, $\Lambda_q^{\mathbf{u}}(\mathbf{M}) := \{\mathbf{r} \in \mathbb{Z}^m | \mathbf{M}\mathbf{r} = \mathbf{u} \bmod q\}$.

Definition 3. Given a parameter $\sigma \in \mathbb{R}^+$, two vectors $\mathbf{c} \in \mathbb{Z}^m$ and $\mathbf{x} \in \mathbb{Z}^m$, we define that the $\forall \mathbf{x} \in \Lambda, \mathcal{D}_{\sigma,\mathbf{c}} = \frac{\rho_{\sigma,\mathbf{c}}(\mathbf{x})}{\rho_{\sigma,\mathbf{c}}(\Lambda)}$ is the discrete Gaussian distribution over lattice Λ, where $\rho_{\sigma,\mathbf{c}}(\mathbf{x}) = \exp(-\pi \frac{\|\mathbf{x}-\mathbf{c}\|^2}{\sigma^2})$ and $\rho_{\sigma,\mathbf{c}}(\Lambda) = \sum_{\mathbf{x} \in \Lambda} \rho_{\sigma,\mathbf{c}}(\mathbf{x})$.

Definition 4. Given two integers $q \geq 2$ and $n \geq 1$, we define a gadget matrix as $\mathbf{G} = \mathbf{I}_n \otimes \mathbf{g}^\top \in \mathbb{Z}^{n \times n\lceil \log q \rceil}$, $\mathbf{g}^\top = [1, 2, \ldots, 2^{\lceil \log q \rceil - 1}] \in \mathbb{Z}_q^{\lceil \log q \rceil}$. Moreover, $\mathbf{G}$ can be extended to a matrix over $\mathbb{Z}_q^{n \times m}$, where $m > n\lceil \log q \rceil$.

Given a vector $\mathbf{s} \in \mathbb{Z}_q^n$, the Learning With Errors (LWE) distribution over $\mathbb{Z}_q^n \times \mathbb{Z}_q$ is sampled by randomly selecting a vector $\mathbf{a} \in \mathbb{Z}_q^n$ and an error vector $e \leftarrow \chi$, where χ is a B-bounded noise distribution s.t. $|e| \leq B$ with non-negligible probability, and returning $(\mathbf{a}, b) = (\mathbf{a}, \mathbf{a}^\top \mathbf{s} + e \bmod q)$.

Definition 5. Given m independent pairs $(\mathbf{a}_i, b_i) \in \mathbb{Z}_q^n \times \mathbb{Z}_q$, where each sample is governed by the following either one to define the decisional $\mathrm{LWE}_{n,m,q,\chi}$ assumption:

1) Pseudo-random sample: $(\mathbf{a}_i, b_i) = (\mathbf{a}_i, \mathbf{a}_i^\top \mathbf{s} + e_i) \in \mathbb{Z}_q^n \times \mathbb{Z}_q$, where $\mathbf{s}$ is a randomly vector, e_i is an error vector, and $\mathbf{a}_i$ is an uniform vector.
2) Random sample: Randomly samples from $\mathbb{Z}_q^n \times \mathbb{Z}_q$.

Moreover, the decisional $\mathrm{LWE}_{n,m,q,\chi}$ assumption has proven to be as hard as the worst-case SIVP and GapSVP according to the reference [47].

Definition 6. [48], [43] Given a positive $\delta > 0$, a function family $F = \{f : \mathbb{Z}_q^d \to \mathbb{Z}_q\}$ and a function $\alpha_F : \mathbb{Z} \to \mathbb{Z}$, we define three evaluation algorithms as follows:

- $\mathbf{M}_f \leftarrow \mathsf{Eval}_{pk}(f, \{\mathbf{M}_i\}_{i=1}^d)$: Given a function $f \in F$ and matrices $\{\mathbf{M}_i\}_{i=1}^d \in \mathbb{Z}_q^{n \times m}$, this algorithm returns a matrix $\mathbf{M}_f \in \mathbb{Z}_q^{n \times m}$.
- $\mathbf{c}_f \leftarrow \mathsf{Eval}_{ct}(f, \{\mathbf{M}_i, \mathbf{c}_i, b_i\}_{i=1}^d\})$: Given a function $f \in F$ and tuples $\{\mathbf{M}_i \in \mathbb{Z}_q^{n \times m}, \mathbf{c}_i \in \mathbb{Z}_q^m, b_i \in \mathbb{Z}_q\}_{i=1}^d$, this algorithm returns a vector $\mathbf{c}_f \in \mathbb{Z}_q^m$, s.t. $\mathbf{c}_f = (\mathbf{M} + f(\mathbf{b})\mathbf{G})^\top \mathbf{s} + \mathbf{e}_f \in \mathbb{Z}_q^m$, where $\mathbf{s} \in \mathbb{Z}_q^n$, $\mathbf{b} = (b_1, b_2, \ldots, b_d)$, $\mathbf{M}_f \leftarrow \mathsf{Eval}_{pk}(f, \{\mathbf{M}_i\}_{i=1}^d)$, $\mathbf{c}_i = (\mathbf{M}_i + b_i\mathbf{G})^\top \mathbf{s} + \mathbf{e}_i$, $\|\mathbf{e}_f\| \leq \Delta$, $\|\mathbf{e}_i\| < \delta$, and $\Delta < \delta\alpha_F(n)$.
- $\mathbf{S}_f \leftarrow \mathsf{Eval}_{sim}(f, \mathbf{M}, \{\mathbf{S}_i, b_i^*\}_{i=1}^d\})$: Given a function $f \in F$, a matrix $\mathbf{M} \in \mathbb{Z}_q^{n \times m}$, and tuples $\{\mathbf{S}_i \in \{-1, 1\}^{m \times m}, b_i^* \in \mathbb{Z}_q\}_{i=1}^d$, this algorithm returns a matrix $\mathbf{S}_f \in \mathbb{Z}_q^{m \times m}$, s.t. $\mathbf{M}\mathbf{S}_f - f(\mathbf{b}^*)\mathbf{G} = \mathbf{M}_f$, where $\mathbf{b}^* = (b_1^*, b_2^*, \ldots, b_d^*)$, $\mathbf{M}_f \leftarrow \mathsf{Eval}_{pk}(f, \{\mathbf{M}\mathbf{S}_i - b_i^*\mathbf{G}\}_{i=1}^d)$, and $\|\mathbf{S}_f\|_2 < \alpha_F(n)$.

For two list $T := (t_1, \ldots, t_d) \in \mathbb{Z}_q^d$, $P := (t'_1, \ldots, t'_\psi) \in \mathbb{Z}_q^\psi$ and a function $f_{t',j} \in F$, we define $f_{t',j}(T) \neq 0$ iff $t'_j \in T$ for $j \in \{1, \ldots, \psi\}$. Otherwise, $f_{t',j}(T) = 0$.

Lemma 1. [49] Given three integers $n, m, q \in \mathbb{Z}$, where $m \geq 2n \log q$, the $\mathsf{TrapGen}(n, m, q)$ algorithm returns a matrix $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$ and a basis $\mathbf{T_A} \in \mathbb{Z}^{m \times m}$, where $\mathbf{A}$ is a uniform matrix and $\|\widetilde{\mathbf{T_A}}\| = \mathcal{O}(\sqrt{n \log q})$.

Lemma 2. [50] Given three integers $n, m, q \in \mathbb{Z}$, two matrices $\mathbf{N}_1, \mathbf{N}_2 \in \mathbb{Z}_q^{n \times m}$, and a basis $\mathbf{T}_{\mathbf{N}_1}$ of $\Lambda_q^{\perp}(\mathbf{N}_1)$, the $\mathsf{ExtendRight}(\mathbf{N}_1, \mathbf{T}_{\mathbf{N}_1}, \mathbf{N}_2)$ algorithm returns a basis $\mathbf{T}_{(\mathbf{N}_1|\mathbf{N}_2)} \in \mathbb{Z}^{2m \times 2m}$ of $\Lambda_q^{\perp}(\mathbf{N}_1|\mathbf{N}_2)$, s.t. $\|\widetilde{\mathbf{T}_{\mathbf{N}_1}}\| = \|\widetilde{\mathbf{T}_{\mathbf{N}_1|\mathbf{N}_2}}\|$.

Lemma 3. [51] Given three integers $n, m, q \in \mathbb{Z}$, four matrices $\mathbf{N}_1, \mathbf{R} \in \mathbb{Z}_q^{n \times m}, \mathbf{N}_2 \in \mathbb{Z}_q^{m \times m}$, $\mathbf{G} \in \mathbb{Z}_q^{n \times m}$, and a basis $\mathbf{T_G}$ of $\Lambda_q^{\perp}(\mathbf{G})$, the $\mathsf{ExtendLeft}(\mathbf{N}_1, \mathbf{G}, \mathbf{T_G}, \mathbf{N}_2)$ algorithm returns a basis $\mathbf{T}_{(\mathbf{N}_1|\mathbf{N}_1\mathbf{N}_2+\mathbf{G})} \in \mathbb{Z}^{2m \times 2m}$ of $\Lambda_q^{\perp}(\mathbf{N}_1|\mathbf{N}_1\mathbf{N}_2 + \mathbf{G})$, s.t. $\|\mathbf{T}_{(\mathbf{N}_1|\mathbf{N}_1\mathbf{N}_2+\mathbf{G})}\| \leq \|\widetilde{\mathbf{T_G}}\|(1 + \|\mathbf{N}_2\|_2)$.

Lemma 4. [50] Given four integers $n, m, m', q \in \mathbb{Z}$, a matrix $\mathbf{M} \in \mathbb{Z}_q^{n \times m}$, a basis $\mathbf{T_M} \in \mathbb{Z}^{m \times m}$ of $\Lambda^{\perp}(\mathbf{M})$, and a parameter $\sigma' \geq \|\widetilde{\mathbf{T_M}}\|\omega(\sqrt{\log m})$, the $\mathsf{RandBasis}(\mathbf{M}, \mathbf{T_M}, \sigma')$ algorithm returns a basis $\mathbf{T}'_{\mathbf{M}} \in \mathbb{Z}^{m \times m}$ of $\Lambda^{\perp}(\mathbf{M})$, s.t. $\|\widetilde{\mathbf{T}'_{\mathbf{M}}}\| \leq \sigma'\sqrt{m}$.

Lemma 5. [50] Given four positive integers $n, m, q, k \in \mathbb{Z}$, where $q \geq 2$, and $m \geq 2n \log q$, a matrix $\mathbf{M} \in \mathbb{Z}_q^{n \times km}$, a basis $\mathbf{T}_{\mathbf{M}_\mathcal{N}}$ of $\Lambda_q^{\perp}(\mathbf{M}_\mathcal{N})$, a set $\mathcal{N} \subseteq [k]$, a vector $\mathbf{u} \in \mathbb{Z}_q^n$, and a parameter $\sigma \geq \|\widetilde{\mathbf{T}_{\mathbf{M}_\mathcal{N}}}\| \cdot \omega(\sqrt{\log km})$, the $\mathsf{GenSamplePre}(\mathbf{M}, \mathbf{T}_{\mathbf{M}_\mathcal{N}}, \mathcal{N}, \mathbf{u}, \sigma)$ algorithm returns a vector $\mathbf{e} \in \mathbb{Z}^{km}$ over $\mathcal{D}_{\Lambda_q^{\mathbf{u}}(\mathbf{M}),\sigma}$, s.t. $\mathbf{M}\mathbf{e} = \mathbf{u} \bmod q$.

Lemma 6. [48] Given five integers $n, m, k, q \in \mathbb{Z}$, a parameter $\sigma > 0$ and two matrices $\mathbf{M} \in \mathbb{Z}_q^{n \times m}$, $\mathbf{U} \in \mathbb{Z}_q^{n \times k}$, if a matrix $\mathbf{K} \in \mathbb{Z}^{m \times k}$ is sampled from $\mathcal{D}_\sigma(\Lambda_q^{\mathbf{U}}(\mathbf{M}))$ and $\mathbf{S}$ is sampled uniformly in $\{-1, 1\}^{m \times m}$, then $\|\mathbf{K}^\top\| \leq \sigma\sqrt{mk}$ and $\|\mathbf{S}^\top\| \leq 20\sqrt{m}$.

Lemma 7. [51] Given a prime $q > 2$, two integers $m > (n + 1) \log q + \omega(\log n)$ and $k = k(n)$, and three matrices $\mathbf{K} \in \mathbb{Z}_q^{n \times m}$, $\mathbf{E} \in \mathbb{Z}_q^{n \times k}$ and $\mathbf{F} \in \{-1, 1\}^{m \times k}$, the distribution $(\mathbf{K}, \mathbf{K}\mathbf{F}, \mathbf{F}^\top \mathbf{r})$ is close to $(\mathbf{K}, \mathbf{E}, \mathbf{F}^\top \mathbf{r})$, $\forall \mathbf{r} \in \mathbb{Z}_q^m$.

4 Problem Formulation

In this section, we present the problem formulation of PunSearch, and the major mathematical notations are defined in Tab. 2.

4.1 System Model

Our PunSearch scheme is composed of four entities, Trusted authority (TA), Data sender, Data receiver, and Mobile cloud server (MCS), as presented in Fig. 2.

- **Trusted authority (TA)** is a *full-trusted* entity, and is responsible for initializing the entire system and calculating the public and initial secret keys of the data receiver from several mobile scenarios. When the TA maintains a punctured tag list $P = \{t_1\}$, it has the ability to generate a punctured secret key promptly

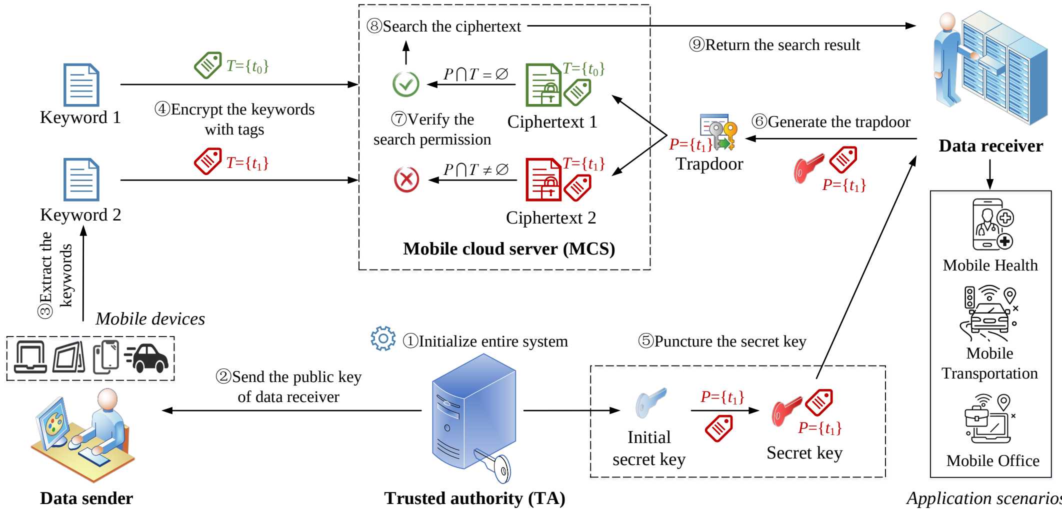


Fig. 2. System model of our PunSearch scheme for mobile cloud. Keyword 1 and Keyword 2 are encrypted with the tags t_1 and t_2, respectively. The initial secret key of the data receiver is punctured with the tag t_1 to generate a search trapdoor, which is only able to search for all the ciphertexts that are not encrypted with t_1, realizing the searchability revocation for a specific keyword.

TABLE 2
Glossary

Notation	Definition
$\mathbf{ck}, \mathbf{tk}$	The keyword
CT	The keyword ciphertext
d	The number of tags
pp	The public parameter
$pk_R, sk_{R,\emptyset}$	The data receiver's public and initial secret keys
P	The punctured tag list, where $P = \{t'_1, \dots, t'_\psi\}$
$sk_{R,\psi-1}$	The secret key with the punctured tag $t'_{\psi-1}$
t	The tag embedded in ciphertext
t'_ψ	The ψ-th punctured tag
T	The tag list, where $T = (t_1, \dots, t_d)$
TD_ψ	The trapdoor with the punctured tag t'_ψ
λ	The security parameter
ψ	The number of punctured tags

based on dynamic mobile data search requirements, which is used to revoke the searchability of this receiver for a specific keyword encrypted with t_1.

- **Mobile cloud server (MCS)** is a *honest-but-curious* entity, so it is designed to store the ciphertext with a tag list T, instead of storing the keyword plaintext directly. After obtaining a trapdoor, the MCS performs the ciphertext search operations if the permission verification is valid (i.e., $P \cap T = \emptyset$).
- **Data sender** is considered *full-trusted*, it can extract the keyword from data files, usually obtained from mobile devices (e.g., laptops, mobile phones, etc.). After receiving a keyword, a data receiver's public key, and a tag list T, the data sender calculates the ciphertext and outsources it to the MCS.
- **Data receiver** is from various mobile scenarios, it is accountable for generating a trapdoor through a keyword together with its secret key, and uploads it to the MCS. As a *full-trusted* entity, if the permission verification is valid and the trapdoor is matched with the corresponding ciphertext, the data receiver will obtain a search result from the MCS.

4.2 Formal Definition

Our PunSearch scheme $\Pi_{\mathrm{PunSearch}}$ includes six algorithms, i.e., **Setup**, **KeyGen**$_R$, **Encrypt**, **Puncture**, **Trapdoor**, **Search**, as described below.

- $pp \leftarrow$ **Setup**(1^λ): The TA inputs a security parameter λ. This algorithm calculates a public parameter pp.
- $(pk_R, sk_{R,\emptyset}) \leftarrow$ **KeyGen**$_R(pp)$: The TA inputs a public parameter pp. This algorithm calculates the public and initial secret keys $(pk_R, sk_{R,\emptyset})$ for a data receiver.
- CT $\leftarrow$ **Encrypt**$(pp, pk_R, \mathbf{ck}, T)$: The data sender inputs a public parameter pp, a public key pk_R of data receiver, a keyword $\mathbf{ck}$, and a tag list T. This algorithm calculates a ciphertext CT with $\mathbf{ck}$ and T.
- $sk_{R,\psi} \leftarrow$ **Puncture**$(pp, sk_{R,\psi-1}, t'_\psi)$: The TA inputs a public parameter pp, a secret key $sk_{R,\psi-1}$ of data receiver with a punctured tag $t'_{\psi-1}$, and a punctured tag t'_ψ. This algorithm calculates a secret key $sk_{R,\psi}$ with a punctured t'_ψ.
- $\mathrm{TD}_\psi \leftarrow$ **Trapdoor**$(pp, pk_R, sk_{R,\psi}, \mathbf{tk})$: The data receiver inputs a public parameter pp, the public key pk_R and secret key $sk_{R,\psi}$ of data receiver with a punctured tag t'_ψ, and a keyword $\mathbf{tk}$. This algorithm calculates a trapdoor TD_ψ with $\mathbf{tk}$ and t'_ψ.
- "Success" or "Failure" $\leftarrow$ **Search**$(pp, \mathrm{CT}, T, \mathrm{TD}_\psi, P)$: The MCS inputs a public parameter pp, a ciphertext CT, a tag list T, and a trapdoor TD_ψ and a punctured tag list P. This algorithm outputs "Success" if the trapdoor meets the permission verification and matches the ciphertext. Otherwise, this algorithm outputs "Failure".

4.3 Security Model

We provide a novel security model for our PunSearch scheme, named PunSearch for ciphertext indistinguishability against chosen keyword attacks (IND-Pun-CKA), which includes interactions of an adversary $\mathcal{A}$ and a challenger $\mathcal{C}$, the specific model $\mathbf{Exp}^{\text{IND-Pun-CKA}}_{\text{PunSearch},\mathcal{A}}(\lambda)$ is defined as follows:

1) **Initialize**: A challenge query index $q^* \in \mathbb{N}^+$, a challenge tag list $T^* = (t_1^*, \ldots, t_d^*)$, the hash function $H : \mathbb{Z}_q^n \to \mathbb{Z}_q^{n\times n}$, and two empty sets P and C are given. For each query q_i, $\mathcal{C}$ maintains a set $\mathcal{Q}$, which is empty initially.
2) **Setup**: Given a security parameter λ, $\mathcal{C}$ invokes the $\mathbf{Setup}(1^\lambda)$ and $\mathbf{KeyGen}_R(pp)$ algorithms to obtain a public parameter pp and the public and initial secret keys $(pk_R, sk_{R,\emptyset})$ of data receiver. Then, $\mathcal{C}$ maintains a set $\mathcal{Q}$ used to record the tuple $(q_i, sk_{R,\psi}, P, C)$. Finally, $\mathcal{C}$ returns pp and pk_R to $\mathcal{A}$.
3) **Phase 1**: $\mathcal{A}$ is responsible for querying these oracles.
   a) Hash Queries $\mathcal{O}_H$: Given a keyword $\mathbf{ck}$ from $\mathcal{A}$, $\mathcal{C}$ keeps a list $\mathcal{H}$ and searches $\mathbf{ck}$ in it, and then sends $H(\mathbf{ck})$ to $\mathcal{A}$.
   b) Ciphertext Queries $\mathcal{O}_{\text{CT}}$: Given a keyword $\mathbf{ck}$, a public key pk_R of data receiver, and a tag list $T = (t_1, \ldots, t_d)$ from $\mathcal{A}$, $\mathcal{C}$ invokes the $\mathbf{Encrypt}(pp, pk_R, \mathbf{ck}, T)$ algorithm to calculate the ciphertext CT, and then transmits it to $\mathcal{A}$.
   c) Puncture Queries $\mathcal{O}_{Pun}$: Given a query index q_i and a punctured tag t'_ψ from $\mathcal{A}$, $\mathcal{C}$ performs the following procedures. If there exists $(q_i, sk_{R,\psi-1}, P, C)$ in $\mathcal{Q}$, $\mathcal{C}$ calls the $\mathbf{Puncture}(pp, sk_{R,\psi-1}, t'_\psi)$ algorithm to calculate the secret key $sk_{R,\psi}$ with the punctured tag t'_ψ, where $P = P \cup \{t'_\psi\}$, and replaces $\{q_i, sk_{R,\psi-1}, P, C\}$ to $\{q_i, sk_{R,\psi}, P, C\}$ in $\mathcal{Q}$. Otherwise, $\mathcal{C}$ invokes $\mathbf{KeyGen}_R(pp)$ and $\mathbf{Puncture}(pp, sk_{R,\emptyset}, t'_\psi)$ to calculate the secret key $sk_{R,\psi}$ with $P = \{t'_\psi\}$, and creates a new tuple $(q_i, sk_{R,\psi}, P, C)$ in $\mathcal{Q}$, where $C = \emptyset$.
   d) Corrupt Queries $\mathcal{O}_{Cor}$: Given a query index q_i from $\mathcal{A}$, $\mathcal{C}$ executes the following procedures:
      - $q_i \neq q^*$: If there exists $(q_i, sk_{R,\psi-1}, P, C)$ in $\mathcal{Q}$, $\mathcal{C}$ sends $sk_{R,\psi-1}$ to $\mathcal{A}$, and assigns $C = P$. Otherwise, $\mathcal{C}$ invokes $\mathbf{KeyGen}_R(pp)$ algorithm to calculate the initial secret key $sk_{R,\emptyset}$, and sends it to $\mathcal{A}$. Then, $\mathcal{C}$ assigns $C = P$, and creates a new tuple $(q_i, sk_{R,\emptyset}, P, C)$ in $\mathcal{Q}$.
      - $q_i = q^*$: If it exists $(q^*, sk_{R,\psi-1}, P, C)$ in $\mathcal{Q}$, $\mathcal{C}$ checks whether $P \cap T^* = \emptyset$. If so, $\mathcal{C}$ sends $\perp$ to $\mathcal{A}$. Otherwise, $\mathcal{C}$ sends the most novel secret key $sk_{R,\psi-1}$ of data receiver to $\mathcal{A}$. If there does not exist $(q^*, sk_{R,\psi-1}, P, C)$ in $\mathcal{Q}$, $\mathcal{C}$ sends $\perp$ to $\mathcal{A}$.

      In subsequent procedures, $\mathcal{C}$ returns $\perp$ for all the $\mathcal{O}_{Cor}$ queries from $\mathcal{A}$.
   e) Trapdoor Queries $\mathcal{O}_{\text{TD}}$: Given a keyword $\mathbf{tk}$ and a public key pk_R of data receiver, $\mathcal{C}$ invokes the $\mathbf{Trapdoor}(pp, pk_R, sk_{R,\psi}, \mathbf{tk})$ to calculate a trapdoor TD_ψ, and then transmits it to $\mathcal{A}$.
4) **Challenge**: $\mathcal{A}$ selects two challenge keywords $\mathbf{ck}_0^*, \mathbf{ck}_1^* \in \mathbb{Z}_q^n$ which have not been queried in **Phase 1**, and transmits them to $\mathcal{C}$. Then, $\mathcal{C}$ chooses a random bit $b \in \{0, 1\}$, and calculates the ciphertext CT_b^* using $\mathbf{Encrypt}(pp, pk_R, \mathbf{ck}_b^*, T^*)$ algorithm. Finally, $\mathcal{C}$ returns it to $\mathcal{A}$.
5) **Phase 2**: $\mathcal{C}$ responds all queries from $\mathcal{A}$ as showed in **Phase 1**, but either $\mathbf{ck}_0^*$ or $\mathbf{ck}_1^*$ cannot be queried in $\mathcal{O}_{\text{CT}}$ and $\mathcal{O}_{\text{TD}}$.
6) **Guess**: $\mathcal{A}$ outputs a bit $b' \in \{0, 1\}$. If $b' = b$, $\mathcal{A}$ wins this game.

The advantage of the adversary $\mathcal{A}$ to win the above-mentioned $\mathbf{Exp}^{\text{IND-Pun-CKA}}_{\text{PunSearch},\mathcal{A}}(\lambda)$ is defined as follows:

$$\mathsf{Adv}^{\text{IND-Pun-CKA}}_{\text{PunSearch},\mathcal{A}}(\lambda) = |\Pr[b' = b] - \frac{1}{2}|.$$

Definition 7. Our PunSearch scheme enjoys IND-Pun-CKA security, if the advantage of a PPT adversary $\mathcal{A}$ to win the above-mentioned $\mathbf{Exp}^{\text{IND-Pun-CKA}}_{\text{PunSearch},\mathcal{A}}(\lambda)$ is negligible.

5 The Design of PunSearch

In this section, we first provide the design rationale of PunSearch and then describe our design in detail. After that, we give the parameter settings and correctness analysis.

5.1 Design Rationale

Traditional lattice-based SE schemes can provide an encrypted search for mobile cloud data sharing. However, they can not support the fine-grained searchability revocation for specific keywords, which is impractical in the mobile cloud. PE is a novel primitive formalized in [39], offering the decryption revocation mechanism through puncturing the secret key with a tag list. Thus, the puncture property is expected to be integrated into the SE scheme to improve its practicality, which motivates this work to design a puncturable encrypted search scheme (abbr. PunSearch).

To achieve it, we introduce the ExtendLeft and RandBasis algorithms to puncture a secret key of the data receiver. As mentioned in Section I, how to generate a search trapdoor from a punctured secret key and realize the encrypted search is our main technical challenge. An intuitive approach is to combine the PE with a lattice-based SE scheme. However, most existing SE schemes invoke the SamplePre or SampleLeft algorithms to generate a search trapdoor. After directly mapping the keyword to a matrix $\mathbf{A_{tk}}$, we are unable to find a suitable basis of $\Lambda_q^\perp(\mathbf{A} \mid \mathbf{A_{tk}} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi}})$ to invoke the SamplePre (or SampleLeft) algorithm, because this punctured secret key is usually a basis of $\Lambda_q^\perp(\mathbf{A} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi}})$. The trapdoor cannot be generated validly, therefore combining the PE and SE primitives directly to design an encrypted search scheme becomes unrealistic.

To address this problem, we first introduce the gadget matrix $\mathbf{G}$ to embed the keyword $\mathbf{tk}$ into a matrix $\mathbf{A_{tk}}$. Then, we generate serval matrices $\mathbf{A}_{f_{t',1}}, \ldots, \mathbf{A}_{f_{t',\psi}}$ with punctured tag list P through evaluation algorithm. After that, we leverage the GenSamplePre technique to sample a vector as the trapdoor, by inputting a matrix $\mathbf{A} \mid \mathbf{A_{tk}} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi}}$ together with a basis of $\Lambda_q^\perp(\mathbf{A} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi}})$ (i.e. a punctured secret key). Till now, we have addressed the trapdoor generation problem. Moreover, we design a novel permission verification method for the search procedure to determine whether this trapdoor has the searchability for a specific ciphertext. In this way, if a secret key has been punctured by a tag t', the trapdoor generated from it can not search the ciphertext with the tag list T. Otherwise, the ciphertext search will be executed to return a result for the data receiver. Consequently, the fine-grained searchability revocation for specific keywords has been realized.

5.2 System Initialization

To begin with, the TA inputs a security parameter 1^λ, and invokes $\mathbf{Setup}(1^\lambda)$ to obtain a public parameter pp, which will be distributed to other entities.

The TA first initializes several integers $n, m, q, d \in \mathbb{Z}$, a parameter σ, and a gadget matrix $\mathbf{G} \in \mathbb{Z}_q^{n\times m}$. Then, the TA selects a vector $\mathbf{u} \in \mathbb{Z}_q^n$ uniformly, and defines a collision-resistant hash function $H : \mathbb{Z}_q^n \to \mathbb{Z}_q^{n\times n}$. Finally, the TA returns the public parameter as $pp := (n, m, q, \sigma, d, \mathbf{G}, \mathbf{u}, H)$, and transmits it to other entities.

5.3 Key Generation

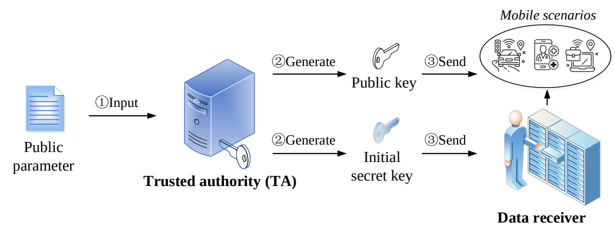


Fig. 3. The procedure of key generation. The TA generates public and initial secret keys and sends them to data receivers from various mobile scenarios.

As shown in Fig. 3, this phase is used to generate the key for the data receiver across various mobile scenarios. After input the public parameter pp, the TA calls the $\mathbf{KeyGen}_R(pp)$ algorithm to obtain the public and initial secret keys $(pk_R, sk_{R,\emptyset})$, and transmits them to the data receiver.

Firstly, the TA invokes $(\mathbf{A}, \mathbf{T_A}) \leftarrow \mathsf{TrapGen}(n, m, q)$ to generate a matrix $\mathbf{A} \in \mathbb{Z}_q^{n\times m}$ and a basis $\mathbf{T_A} \in \mathbb{Z}^{m\times m}$ of lattice $\Lambda_q^\perp(\mathbf{A})$. After that, it selects $d+1$ random matrices $\mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_d \in \mathbb{Z}_q^{n\times m}$ used to calculate the ciphertext. The public and initial secret keys are defined as:

$$pk_R := (\mathbf{A}, \mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_d), sk_{R,\emptyset} := \mathbf{T_A}.$$

Ultimately, the TA transmits the key pair $(pk_R, sk_{R,\emptyset})$ to the data receiver from various mobile scenarios through a confidential channel.

5.4 Ciphertext Generation

The data owner selects a keyword $\mathbf{ck} \in \mathbb{Z}_q^n$ from the data uploaded by the mobile devices (e.g., laptop, pad, mobile phone, smart vehicle). Before generating the ciphertext, the data owner initially defines a tag list $T := (t_1, \ldots, t_d)$ and uses the public parameter pp, the data receiver's public key pk_R, and the keyword $\mathbf{ck}$ as inputs to execute the $\mathbf{Encrypt}(pp, pk_R, \mathbf{ck}, T)$ algorithm, generating keyword ciphertext CT with T. The procedure is shown in Fig. 4.

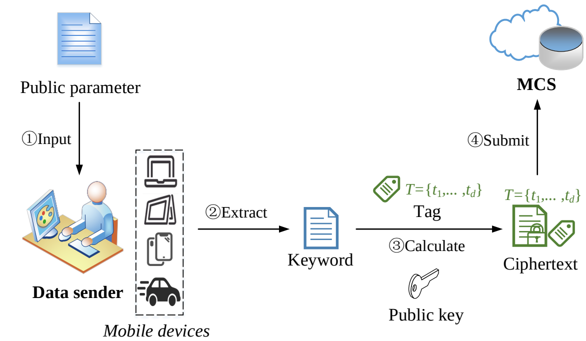


Fig. 4. The procedure of ciphertext generation. The data sender extracts keywords from the data in the mobile devices, encrypts them with a tag list T to obtain the ciphertext, and then uploads it to the MCS.

First of all, the data sender calculates a matrix $\mathbf{A_{ck}} = \mathbf{A}_0 + H(\mathbf{ck})\mathbf{G} \in \mathbb{Z}_q^{n\times m}$ to embed the keyword $\mathbf{ck}$. Subsequently, the data sender selects several vectors $\mathbf{s} \in \mathbb{Z}_q^n$ and $\mathbf{e}_0 \leftarrow \chi^m$, a value $e_2 \leftarrow \chi$, and many matrices $\mathbf{R_{ck}}, \mathbf{R}_{t,1}, \ldots, \mathbf{R}_{t,d} \in \{-1, 1\}^{m\times m}$, and calculates a matrix $\mathbf{A}_{\mathbf{ck},T} = (\mathbf{A} \mid \mathbf{A_{ck}} \mid \mathbf{A}_1 + t_1\mathbf{G} \mid \ldots \mid \mathbf{A}_d + t_d\mathbf{G}) \in \mathbb{Z}_q^{n\times(d+2)m}$ for the tag list $T = (t_1, \ldots, t_d)$. After that, the data sender calculates

$$\mathbf{c}_1 = \mathbf{A}_{\mathbf{ck},T}^\top \mathbf{s} + \mathbf{e}_1 \in \mathbb{Z}_q^{(d+2)m}, c_2 = \mathbf{u}^\top \mathbf{s} + e_2 \in \mathbb{Z}_q,$$

where the error term $\mathbf{e}_1$ as

$$\begin{aligned} \mathbf{e}_1 &= (\mathbf{I}_m \mid \mathbf{R_{ck}} \mid \mathbf{R}_{t,1} \mid \ldots \mid \mathbf{R}_{t,d})^\top \mathbf{e}_0 \\ &= (\mathbf{e}_0^\top \mid (\mathbf{R}_{\mathbf{ck}}^\top \mathbf{e}_0)^\top \mid (\mathbf{R}_{t,1}^\top \mathbf{e}_0)^\top \mid \ldots \mid (\mathbf{R}_{t,d}^\top \mathbf{e}_0)^\top)^\top \\ &:= (\mathbf{e}_0^\top \mid \mathbf{e}_{\mathbf{ck}}^\top \mid \mathbf{e}_{t,1}^\top \mid \ldots \mid \mathbf{e}_{t,d}^\top)^\top \in \mathbb{Z}_q^{(d+2)m}. \end{aligned}$$

In the end, the data sender defines the ciphertext $\mathrm{CT} := (\mathbf{c}_1, c_2)$ with the keyword $\mathbf{ck}$ and the tag list T, and then outsources it to the MCS.

5.5 Puncture Phase

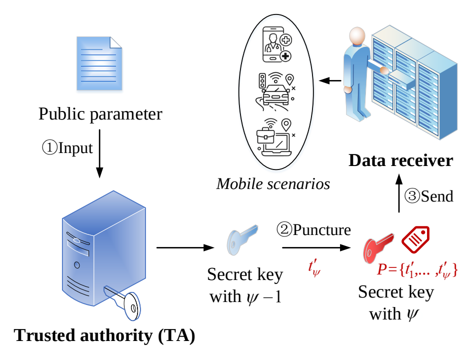


Fig. 5. The procedure of key puncture. The TA punctures the secret key of the data receiver from the mobile scenarios with the tag t'_ψ to generate a new punctured secret key.

As illustrated in Fig. 5, the puncture phase is dedicated to generating a new secret key with a punctured tag for the data receiver from various mobile scenarios. The TA maintains a list P used to record the punctured tags. After input a public parameter pp, a secret key $sk_{R,\psi-1}$ with the punctured tag t'_ψ, the TA calls $\mathbf{Puncture}(pp, sk_{R,\psi-1}, t'_\psi)$ algorithm, and then outputs a secret key $sk_{R,\psi}$ with the punctured tag t'_ψ. Specifically, this phase has been divided into two cases.

5.5.1 If $P = \emptyset$

The TA evaluates $\mathbf{A}_{f_{t',1}} \leftarrow \mathsf{Eval}_{pk}(\{\mathbf{A}_i\}_{i=1}^{d}, f_{t',1})$ to generate a matrix $\mathbf{A}_{f_{t',1}} \in \mathbb{Z}_q^{n\times m}$. Subsequently, the TA invokes ExtendRight and RandBasis algorithm to generate and randomize a basis as

$$\mathbf{T}_{t',1} \leftarrow \mathsf{ExtendRight}(\mathbf{A}, \mathbf{T}_{\mathbf{A}}, \mathbf{A}_{f_{t',1}}),$$
$$\widehat{\mathbf{T}_{t',1}} \leftarrow \mathsf{RandBasis}(\mathbf{A} \mid \mathbf{A}_{f_{t',1}}, \mathbf{T}_{t',1}, \sigma_1),$$

where $\mathbf{T}_{t',1} \in \mathbb{Z}^{2m\times 2m}$ is a basis of lattice $\Lambda_q^{\perp}(\mathbf{A} \mid \mathbf{A}_{f_{t',1}})$, $\widehat{\mathbf{T}_{t',1}} \in \mathbb{Z}^{2m\times 2m}$ is a randomizing basis of lattice $\Lambda_q^{\perp}(\mathbf{A} \mid \mathbf{A}_{f_{t',1}})$ generated from $\mathbf{T}_{t',1}$, and $\sigma_1 = \omega(\alpha_F(n)\sqrt{\log m})$. As a result, the TA updates the punctured list $P = P \cup \{t'_1\}$, and then return a new secret key $sk_{R,1} = \widehat{\mathbf{T}_{t',1}}$ with the punctured tag t'_1 to the data receiver.

5.5.2 If $P \neq \emptyset$

Assume that $P = \{t'_1, \ldots, t'_{\psi-1}\}$, the TA evaluates $\mathbf{A}_{f_{t',\psi}} \leftarrow \mathsf{Eval}_{pk}(f_{t',\psi}, \{\mathbf{A}_i\}_{i=1}^{d})$ to generate a matrix $\mathbf{A}_{f_{t',\psi}} \in \mathbb{Z}_q^{n\times m}$. The ExtendRight and RandBasis algorithms are invoked by the TA to generate a basis as

$$\mathbf{T}_{t',\psi} \leftarrow \mathsf{ExtendRight}(\mathbf{A} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi-1}}, \widehat{\mathbf{T}_{t',\psi-1}}, \mathbf{A}_{f_{t',\psi}}),$$
$$\widehat{\mathbf{T}_{t',\psi}} \leftarrow \mathsf{RandBasis}(\mathbf{A} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi}}, \mathbf{T}_{t',\psi}, \sigma_\psi),$$

where $\mathbf{T}_{t',\psi} \in \mathbb{Z}^{(\psi+1)m\times(\psi+1)m}$ is a basis of lattice $\Lambda_q^{\perp}(\mathbf{A} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi}})$, $\widehat{\mathbf{T}_{t',\psi}} \in \mathbb{Z}^{(\psi+1)m\times(\psi+1)m}$ is a randomizing basis of lattice $\Lambda_q^{\perp}(\mathbf{A} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi}})$ generated from $\mathbf{T}_{t',\psi}$, and $\sigma_\psi = \sigma_1(\sqrt{m\log m})^{\psi-1}$. Eventually, the TA updates the punctured list $P = P \cup \{t'_\psi\}$, and then sends a new secret key $sk_{R,\psi} = \widehat{\mathbf{T}_{t',\psi}}$ with the punctured tag t'_ψ to the data receiver. For $\forall t' \in P = \{t'_1, \ldots, t'_\psi\}$, the trapdoor calculated by $sk_{R,1}$ (or $sk_{R,\psi}$) is unable to search for ciphertext with the tag list T if $t' \in T$, which can revoke this trapdoor's searchability for specific keywords.

5.6 Trapdoor Generation

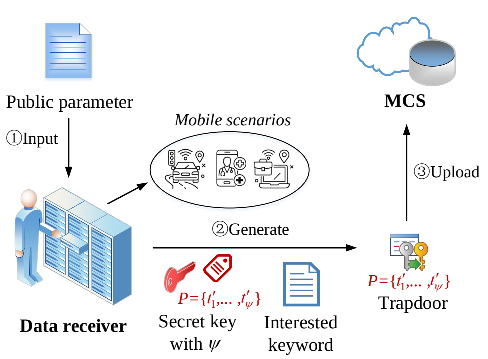


Fig. 6. The procedure of trapdoor generation. The data receiver from mobile scenarios utilizes its secret key to generate a search trapdoor with a puncture list P, and then uploads it to the MCS.

In Fig. 6, to generate a trapdoor utilized own secret key $sk_{R,\psi}$ with a punctured tag t'_ψ, the data receiver from different mobile scenarios has the ability to call **Trapdoor**$(pp, pk_R, sk_{R,\psi}, \mathbf{tk})$ algorithm after inputting the public parameter pp, the public and secret keys $(pk_R, sk_{R,\psi})$ and a keyword $\mathbf{tk} \in \mathbb{Z}_q^n$ to be searched. The specific procedure is as follows.

At the beginning, the data receiver calculates a matrix $\mathbf{A}_{\mathbf{tk}} = \mathbf{A}_0 + H(\mathbf{tk})\mathbf{G} \in \mathbb{Z}_q^{n\times m}$ to embed the keyword $\mathbf{tk}$. For $j \in P = \{t'_1, \ldots, t'_\psi\}$, $\mathbf{A}_{f_{t',j}} \leftarrow \mathsf{Eval}_{pk}(\{\mathbf{A}_i\}_{i=1}^{d}, f_{t',j})$ is evaluated by the data receiver to generate a matrix $\mathbf{A}_{f_{t',j}} \in \mathbb{Z}_q^{n\times m}$. Following that, the data receiver samples a vector $\mathbf{td}_\psi \in \mathbb{Z}_q^{(\psi+2)m}$ as

$$\mathbf{td}_\psi \leftarrow \mathsf{GenSamplePre}(\mathbf{A} \mid \mathbf{A}_{\mathbf{tk}} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi}}, \widehat{\mathbf{T}_{t',\psi}}, \{1, 3, \ldots, \psi+2\}, \mathbf{u}, \sigma),$$

such that $(\mathbf{A} \mid \mathbf{A}_{\mathbf{tk}} \mid \mathbf{A}_{f_{t',1}} \mid \ldots \mid \mathbf{A}_{f_{t',\psi}})\mathbf{td}_\psi = \mathbf{u} (\mod q)$.

At last, the data receiver uploads this trapdoor $\mathrm{TD}_\psi := \mathbf{td}_\psi$ with the keyword $\mathbf{tk}$ and the punctured tag t'_ψ to the MCS.

5.7 Search Phase

For each ciphertext CT with the tag list T, the MCS inputs the parameter pp, a trapdoor TD_ψ with the punctured tag t'_ψ, and calls the **Search**$(pp, \mathrm{CT}, T, \mathrm{TD}_\psi, P)$ algorithm to execute two-level procedures and then finds the search results as in Fig. 7.

5.7.1 The permission verification

For the punctured tag list P, if $P \cap T \neq \emptyset$, i.e., if there exists $j \in \{1, \ldots, \psi\}$, such that $t'_j \in T \iff f_{t',j}(T) \neq 0$, this algorithm outputs "Failure" to the data receiver, meaning that the trapdoor does not have the searchability for the keyword CT. Otherwise, the MCS executes the following procedure.

5.7.2 The ciphertext search

Parse $\mathrm{CT} = (\mathbf{c}_1, c_2)$, where $\mathbf{c}_1 = (\mathbf{c}^\top \mid \mathbf{c}_{\mathbf{ck}}^\top \mid \mathbf{c}_{t,1}^\top \mid \ldots \mid \mathbf{c}_{t,d}^\top)^\top$. For $j \in \{1, \ldots, \psi\}$, the MCS evaluates Eval_{ct} algorithm to generate a vector $\mathbf{c}_{f_{t',j}} \in \mathbb{Z}_q^m$ as

$$\mathbf{c}_{f_{t',j}} \leftarrow \mathsf{Eval}_{ct}(f_{t',j}, \{\mathbf{A}_i, \mathbf{c}_i, t_i\}_{i=1}^{d}).$$

After that, the MCS calculates a vector $\mathbf{c}'_1 = (\mathbf{c}^\top \mid \mathbf{c}_{\mathbf{ck}}^\top \mid \mathbf{c}_{f_{t',1}}^\top \mid \ldots \mid \mathbf{c}_{f_{t',\psi}}^\top)^\top \in \mathbb{Z}_q^{(\psi+2)m}$, and computes a value as

$$\eta = c_2 - \mathbf{td}_\psi^\top \mathbf{c}'_1.$$

If $|\eta| < \lfloor \frac{q}{4} \rfloor$, this algorithm outputs "Success" to the data receiver, which means that the ciphertext CT and the trapdoor TD_ψ correspond to the same keyword. Otherwise, this algorithm outputs "Failure" to the data receiver.

5.8 Correctness Analysis and Parameters Setting

Given a tag list $T = (t_1, \ldots, t_d)$ and a punctured list $P = \{t'_1, \ldots, t'_\psi\}$, assume that a data receiver's public key is $pk_R = (\mathbf{A}, \mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_d)$, a ciphertext $\mathrm{CT} = (\mathbf{c}_1, c_2)$ with $\mathbf{ck}$ and a trapdoor is $\mathrm{TD}_\psi = \mathbf{td}_\psi$ with $\mathbf{tk}$.

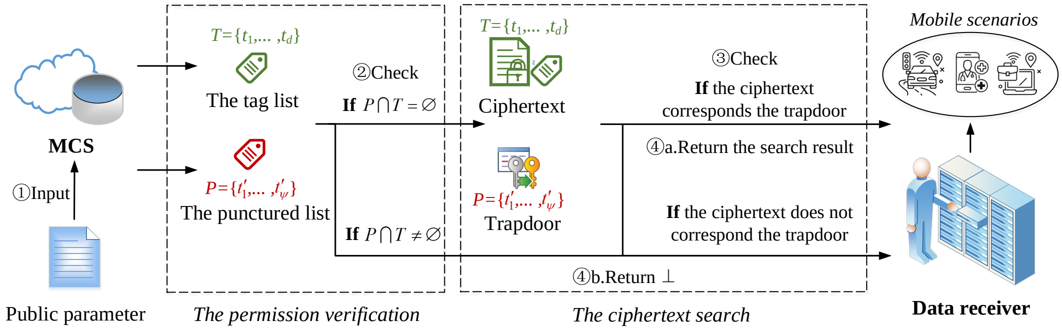


Fig. 7. The procedure of the search phase. The search phase consists of two steps. In the permission verification step, the MCS determines whether they have the searchability for this ciphertext by checking the condition $P \cap T \stackrel{?}{=} \emptyset$. If this condition is satisfied, the MCS proceeds to the ciphertext search, where it is verified whether the search trapdoor corresponds to the keyword ciphertext.

If $f_{t',j}(T) = 0$ where $j \in \{1, \dots, \psi\}$ and $\mathbf{ck} = \mathbf{tk}$:

$$\begin{aligned}\eta &= c_2 - \mathbf{td}_\psi^\top \mathbf{c}_1' = \mathbf{u}^\top \mathbf{s} + e_2 - \mathbf{td}_\psi^\top (\mathbf{c}^\top \mid \mathbf{c}_{\mathbf{ck}}^\top \mid \mathbf{c}_{f_{t',1}}^\top \mid \dots \mid \mathbf{c}_{f_{t',\psi}}^\top)^\top \\ &= \mathbf{u}^\top \mathbf{s} + e_2 - \mathbf{td}_\psi^\top ((\mathbf{A}^\top \mathbf{s} + \mathbf{e}_0)^\top \mid (\mathbf{A}_{\mathbf{ck}}^\top \mathbf{s} + \mathbf{e}_{\mathbf{ck}})^\top \mid ((\mathbf{A}_{f_{t',1}} \\ &\quad + f_{t',1}(T)\mathbf{G})^\top \mathbf{s} + \mathbf{e}_{f_{t',1}})^\top \mid \dots \mid ((\mathbf{A}_{f_{t',\psi}} + f_{t',\psi}(T)\mathbf{G})^\top \mathbf{s} \\ &\quad + \mathbf{e}_{f_{t',\psi}})^\top)^\top \\ &= \mathbf{u}^\top \mathbf{s} + e_2 - ((\mathbf{A} \mid \mathbf{A}_{\mathbf{ck}} \mid \mathbf{A}_{f_{t',1}} \mid \dots \mid \mathbf{A}_{f_{t',\psi}})\mathbf{td}_\psi)^\top \mathbf{s} \\ &\quad - \mathbf{td}_\psi^\top (\mathbf{e}_0^\top \mid \mathbf{e}_{\mathbf{ck}}^\top \mid \mathbf{e}_{f_{t',1}}^\top \mid \dots \mid \mathbf{e}_{f_{t',\psi}}^\top)^\top \\ &= \mathbf{u}^\top \mathbf{s} + e_2 - \mathbf{u}^\top \mathbf{s} - \mathbf{td}_\psi^\top (\mathbf{e}_0^\top \mid \mathbf{e}_{\mathbf{ck}}^\top \mid \mathbf{e}_{f_{t',1}}^\top \mid \dots \mid \mathbf{e}_{f_{t',\psi}}^\top)^\top \\ &= e_2 - \mathbf{td}_\psi^\top (\mathbf{e}_0^\top \mid \mathbf{e}_{\mathbf{ck}}^\top \mid \mathbf{e}_{f_{t',1}}^\top \mid \dots \mid \mathbf{e}_{f_{t',\psi}}^\top)^\top.\end{aligned}$$

Then, we have that:

$$\begin{aligned}&\|\eta\| \le \|e_2\| + \|\mathbf{td}_\psi^\top\| \cdot \|(\mathbf{e}_0^\top \mid \mathbf{e}_{\mathbf{ck}}^\top \mid \mathbf{e}_{f_{t',1}}^\top \mid \dots \mid \mathbf{e}_{f_{t',\psi}}^\top)^\top\| \\ &\le B + \sigma\sqrt{(\psi+2)m}(B + 20\sqrt{m}B + \psi\alpha_F(n)B).\end{aligned}$$

Based on this, we provide the parameter settings as:

- $B \ge \sqrt{n}\omega(\log n)$ for LWE assumption.
- $m \ge \lceil 2n \log q \rceil$ for TrapGen lemma.
- $\alpha_F(n) > \sqrt{n \log m}$ for evaluation algorithms.
- $\sigma_1 = \omega(\alpha_F(n)\sqrt{\log m})$ and $\{\sigma_i = \sigma_1(\sqrt{m \log m})^{i-1}\}_{i=2}^{\psi}$ for ExtendRight and RandBasis lemmas.
- $\sigma \ge (\psi+1)m \cdot \omega(\log(\psi+1)m)$ for GenSamplePre lemma.
- $B + \sigma\sqrt{(\psi+2)m}(B + 20\sqrt{m}B + \psi\alpha_F(n)B) < \frac{q}{4}$ for correctness.

6 Security Analysis

In this section, we prove the IND-Pun-CKA security of our PunSearch scheme.

Theorem 1. If the $\mathrm{LWE}_{n,m,q,\chi}$ assumption holds, our PunSearch scheme satisfies IND-Pun-CKA security in the ROM. For any PPT adversary $\mathcal{A}$, if $\mathcal{A}$ disrupts our scheme with a non-negligible advantage ϵ, then we can build a PPT challenger $\mathcal{C}$ to address the $\mathrm{LWE}_{n,m,q,\chi}$ assumption with a non-negligible probability.

Proof Given a challenge query index $q^* \in \mathbb{N}^+$, a challenge tag list $T^* = (t_1^*, \dots, t_d^*)$, the hash function $H : \mathbb{Z}_q^n \to \mathbb{Z}_q^{n \times n}$, and two empty sets P and C. For each query q_i, $\mathcal{C}$ maintains a set $\mathcal{Q}$, which is empty initially.

$\widehat{\textbf{Game 0}}$: This is equivalent to the IND-Pun-CKA security model defined in Section IV. C. Specifically, $\mathcal{C}$ invokes $\textbf{Setup}(1^\lambda)$ algorithm to initialize this system, and responds to all the queries from $\mathcal{A}$. Then, $\mathcal{A}$ chooses two challenge keyword $\mathbf{ck}_0^*, \mathbf{ck}_1^* \in \mathbb{Z}_q^n$ which have not been queried in **Phase 1**, and transmits them to $\mathcal{C}$. After that, $\mathcal{C}$ selects a bit $b \in \{0,1\}$, invokes $\textbf{Encrypt}(pp, pk_R, \mathbf{ck}_b^*, T^*)$ algorithm to compute a challenge ciphertext CT_b, and then returns to $\mathcal{A}$. Finally, $\mathcal{A}$ outputs $b' \in \{0,1\}$ and wins this game if $b' = b$.

$\widehat{\textbf{Game 1}}$: The $\widehat{\textbf{Game 1}}$ is equivalent to $\widehat{\textbf{Game 0}}$, except that the calculation way of $\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_d$. In $\widehat{\textbf{Game 1}}$, $\mathcal{C}$ selects many matrices $\mathbf{R}_{\mathbf{ck}}^*, \mathbf{R}_{t,1}^*, \dots, \mathbf{R}_{t,d}^* \in \{-1,1\}^{m \times m}$ and $\mathbf{h}^* \leftarrow \mathbb{Z}_q^{n \times n}$ randomly, and then calculates $\mathbf{A}_0 = \mathbf{A}\mathbf{R}_{\mathbf{ck}}^* - \mathbf{h}^*\mathbf{G}$, $\mathbf{A}_1 = \mathbf{A}\mathbf{R}_{t,1}^* - t_1^*\mathbf{G}, \dots, \mathbf{A}_d = \mathbf{A}\mathbf{R}_{t,d}^* - t_d^*\mathbf{G}$. According to Lemma 7, $\mathbf{A}\mathbf{R}_{\mathbf{ck}}^*, \mathbf{A}\mathbf{R}_{t,1}^*, \dots, \mathbf{A}\mathbf{R}_{t,d}^*$ are indistinguishable with uniform distribution. Consequently, $\widehat{\textbf{Game 0}}$ and $\widehat{\textbf{Game 1}}$ cannot be distinguished statistically. Based on above-mentioned settings, when $\mathcal{A}$ executes the queries in **Phase 1**, $\mathcal{C}$ executes the following procedures to obtain the answers:

- Hash Queries $\mathcal{O}_H$: Assume q_H represents the maximum number of hash queries performed by $\mathcal{A}$, and $\mathcal{C}$ selects a random value $\omega^* \in [q_H]$ and maintains a list $\mathcal{H}$. For $i \in [q_H]$, $\mathcal{A}$ submits a keyword $\mathbf{ck}_i$ to query its hash value $H(\mathbf{ck}_i)$. If $i = \omega^*$, C sets $H(\mathbf{ck}_i) = \mathbf{h}^*$, updates $\mathcal{H} = \mathcal{H} \cup \{\mathbf{ck}_i, H(\mathbf{ck}_i)\}$, and returns $H(\mathbf{ck}_i)$ to $\mathcal{A}$. Otherwise, if there exists $\{\mathbf{ck}_i, H(\mathbf{ck}_i)\}$ in $\mathcal{H}$, $\mathcal{C}$ sends $H(\mathbf{ck}_i)$ to $\mathcal{A}$. Otherwise, $\mathcal{C}$ selects a random matrix $\mathbf{h} \leftarrow \mathbb{Z}_q^{n \times n}$ which is not included in $\mathcal{H}$, sets $H(\mathbf{ck}_i) = \mathbf{h}$, updates $\mathcal{H} = \mathcal{H} \cup \{\mathbf{ck}_i, H(\mathbf{ck}_i)\}$, and returns $H(\mathbf{ck}_i)$ to $\mathcal{A}$.
- Ciphertext Queries $\mathcal{O}_{\mathrm{CT}}$: After receiving a keyword $\mathbf{ck}$ and a tag list $T = (t_1, \dots, t_d)$ from $\mathcal{A}$, $\mathcal{C}$ calls $\mathrm{CT} \leftarrow \textbf{Encrypt}(pp, pk_R, \mathbf{ck}, T)$ to calculate a ciphertext CT with $\mathbf{ck}$ and T, and transmits it to $\mathcal{A}$.
- Puncture Queries $\mathcal{O}_{Pun}$: After obtaining a query index q_i and a punctured tag t'_ψ from $\mathcal{A}$, $\mathcal{C}$ proceeds as follow.
  - $q_i \ne q^*$: If there exists $(q_i, sk_{R,\psi-1}, P, C)$ in $\mathcal{Q}$, $\mathcal{C}$ calls $sk_{R,\psi} \leftarrow \textbf{Puncture}(pp, sk_{R,\psi-1}, t'_\psi)$ to obtain the secret key $sk_{R,\psi}$ with the punctured tag t'_ψ, where $P = P \cup \{t'_\psi\}$, and replaces $\{q_i, sk_{R,\psi-1}, P, C\}$ to $\{q_i, sk_{R,\psi}, P, C\}$ in $\mathcal{Q}$. Otherwise, $\mathcal{C}$ invokes $(\mathbf{A}, \mathbf{T}_{\mathbf{A}}) \leftarrow \mathsf{TrapGen}(n, m, q)$ to obtain the initial

secret key $sk_{R,\emptyset} = \mathbf{T_A}$ of data receiver. Then, $\mathcal{C}$ calls $sk_{R,\psi} \leftarrow$ **Puncture**$(pp, sk_{R,\emptyset}, t'_\psi)$, and sets $P = \{t'_\psi\}$ and $C = \emptyset$. For further queries, $\mathcal{C}$ calls **Puncture** algorithm accordingly, and returns $sk_{R,\psi-1}$ to $\mathcal{A}$.

– $q_i = q^*$: If there exists $(q^*, -, P, C)$, $\mathcal{C}$ sets $P = P \cup \{t'_\psi\}$, and replaces $(q^*, -, P, C)$ to the new tuple. Otherwise, $\mathcal{C}$ sets $P = P \cup \{t'_\psi\}$ and $C = \emptyset$, and constructs $(q^*, -, P, C)$ as a new tuple. In this circumstance, $\mathcal{C}$ does not generate the punctured secret key to $\mathcal{A}$.

- Corrupt Queries $\mathcal{O}_{Cor}$: After receiving a query index q_i from $\mathcal{A}$, $\mathcal{C}$ executes the following procedures.
  – $q_i \neq q^*$: If there exists $(q_i, sk_{R,\psi-1}, P, C)$, $\mathcal{C}$ returns $sk_{R,\psi-1}$ to $\mathcal{A}$, and assigns $C = P$. Otherwise, $\mathcal{C}$ sends the initial secret key $sk_{R,\emptyset}$ to $\mathcal{A}$, and assigns $C = P = \emptyset$. In subsequent procedures, $\mathcal{C}$ returns $\perp$ for all the queries from $\mathcal{A}$.
  – $q_i = q^*$: If there exists $(q^*, -, P, C)$, $\mathcal{C}$ checks if $P \cap T^* = \emptyset$. If $P \cap T^* = \emptyset$, $\mathcal{C}$ returns $\perp$ to $\mathcal{A}$. Otherwise, it means that there exists a punctured tag $t'_k \in P$, such that $f_{t',k}(T^*) \neq 0$. We assume that $P = \{t'_1, \ldots, t'_k\}$, if $k > 1$, the t_1 and t_k need to be swapped to construct $P = \{t'_k, t'_2, \ldots, t'_{k-1}, t'_1\}$. Then, $\mathcal{C}$ calculates $\mathbf{R}^*_{f_{t',k}} \leftarrow \mathsf{Eval}_{sim}(f_{t',k}, \mathbf{A}, \{\mathbf{R}^*_j, t^*_j\}_{j=1}^d)$, and sets $\mathbf{A}_{f_{t',k}} = \mathbf{A}\mathbf{R}^*_{f_{t',k}} - f_{t',k}(T^*)\mathbf{G}$. After that, $\mathcal{C}$ invokes $\mathbf{T}_{\mathbf{A}|\mathbf{A}_{f_{t',k}}} \leftarrow \mathsf{ExtendLeft}(\mathbf{A}, -f_{t',k}(T^*)\mathbf{G}, \mathbf{T_G}, \mathbf{R}^*_{f_{t',k}})$, $\mathbf{T}_{t',k} \leftarrow \mathsf{ExtendRight}(\mathbf{A} \mid \mathbf{A}_{f_{t',k}}, \mathbf{T}_{\mathbf{A}|\mathbf{A}_{f_{t',k}}}, \mathbf{A}_{f_{t',2}} \mid \ldots \mid \mathbf{A}_{f_{t',k-1}} \mid \mathbf{A}_{f_{t',1}})$ and $\widehat{\mathbf{T}_{t',k}} \leftarrow \mathsf{RandBasis}(\mathbf{A} \mid \mathbf{A}_{f_{t',k}} \mid \mathbf{A}_{f_{t',2}} \mid \ldots \mid \mathbf{A}_{f_{t',k-1}} \mid \mathbf{A}_{f_{t',1}}, \mathbf{T}_{t',k}, \sigma_k)$. Finally, $\mathcal{C}$ outputs $sk_{R,\psi} = \widehat{\mathbf{T}_{t',k}}$ to $\mathcal{A}$. Otherwise, $\mathcal{C}$ sets $P = \emptyset$, and outputs $\perp$ to $\mathcal{A}$. In subsequent procedures, $\mathcal{C}$ returns $\perp$ for all the queries from $\mathcal{A}$.
- Trapdoor Queries $\mathcal{O}_{\mathrm{TD}}$: After receiving a query index q_i, a keyword $\mathbf{tk}$ and a public key pk_R of data receiver from $\mathcal{A}$, $\mathcal{C}$ executes the following procedures.
  – $q_i \neq q^*$: $\mathcal{C}$ obtains the secret key $sk_{R,\psi}$ using the same way in $\mathcal{O}_{Pun}$, calls **Trapdoor**$(pp, pk_R, sk_{R,\psi}, \mathbf{tk})$ algorithm to obtain a trapdoor $\mathrm{TD}_\psi = \mathbf{td}_\psi$, and transmits it to $\mathcal{A}$.
  – $q_i = q^*$: If $H(\mathbf{tk}) = \mathbf{h}^*$, $\mathcal{C}$ aborts this game, and the probability of abort is at most $\frac{1}{q_H}$. We assume that there exists a query index $\omega^* \in [q_H]$, due to the collision-resistance property of H, for the query $i \in [q_H] \subset \{\omega^*\}$, we can hold that $H(\mathbf{tk}) \neq \mathbf{h}^*$. Otherwise, for $P = \{t'_k, t'_2, \ldots, t'_{k-1}, t'_1\}$, $\mathcal{C}$ invokes $\mathbf{td}_k \leftarrow \mathsf{GenSamplePre}(\mathbf{A} \mid \mathbf{A_{tk}} \mid \mathbf{A}_{f_{t',k}} \mid \ldots \mid \mathbf{A}_{f_{t',1}}, \widehat{\mathbf{T}_{t',k}}, \{1, 3, \ldots, k+2\}, \sigma)$ to generate the trapdoor $\mathbf{td}_k$. Finally, $\mathcal{C}$ returns $\mathrm{TD}_\psi = \mathbf{td}_k$ to $\mathcal{A}$.

$\widehat{\textbf{Game 2}}$: The $\widehat{\textbf{Game 2}}$ is equivalent to $\widehat{\textbf{Game 1}}$, except that the calculation way of the challenge ciphertext CT^*. In $\widehat{\textbf{Game 2}}$, $\mathcal{C}$ selects CT^* from $\mathbb{Z}^{(d+2)m} \times \mathbb{Z}_q$ randomly. Thus, $\mathcal{A}$ can not have the advantage in $\widehat{\textbf{Game 2}}$.

Reduction to LWE: Assume that an adversary $\mathcal{A}$ can distinguish $\widehat{\textbf{Game 1}}$ and $\widehat{\textbf{Game 2}}$ with non-negligible probability, we can construct an algorithm $\mathcal{B}$ to solve $\mathrm{LWE}_{n,m,q,\chi}$ assumption with non-negligible probability.

- **LWE instance**: $\mathcal{B}$ initializes two LWE instances $(\mathbf{A}, \mathbf{c_A}) \in \mathbb{Z}_q^{n\times m} \times \mathbb{Z}_q^m$, and $(\mathbf{u}, c_\mathbf{u}) \in \mathbb{Z}_q^n \times \mathbb{Z}_q$, which are either random (i.e. $\mathbf{c_A} \xleftarrow{\$} \mathbb{Z}_q^m$, $c_\mathbf{u} \xleftarrow{\$} \mathbb{Z}_q$) or pseudorandom (i.e. satisfying $\mathbf{c_A} = \mathbf{A}^\top\mathbf{s} + \mathbf{e}_0$, $c_\mathbf{u} = \mathbf{u}^\top\mathbf{s} + e_2$), where $\mathbf{s} \in \mathbb{Z}_q^n$, $\mathbf{e}_0 \in \chi^m$, and $e_2 \in \chi$. After receiving the answers from $\mathcal{A}$, $\mathcal{B}$ needs to distinguish these two aforementioned cases.
- **Initialize**: A challenge query index $q^* \in \mathbb{N}^+$, a challenge tag list $T^* = (t^*_1, \ldots, t^*_d)$, the hash function $H : \mathbb{Z}_q^n \to \mathbb{Z}_q^{n\times n}$, and two empty sets P and C are given. For each query q_i, $\mathcal{C}$ maintains a set $\mathcal{Q}$, which is empty initially.
- **Setup**: $\mathcal{B}$ invokes $(\mathbf{A}, \mathbf{T_A}) \leftarrow \mathsf{TrapGen}(n, m, q)$ to obtain a matrix $\mathbf{A} \in \mathbb{Z}_q^{n\times m}$ and a basis $\mathbf{T_A} \in \mathbb{Z}^{m\times m}$ firstly. Then, $\mathcal{B}$ chooses many matrices $\mathbf{R}^*_{\mathbf{ck}}, \mathbf{R}^*_{t,1}, \ldots, \mathbf{R}^*_{t,d} \in \{-1,1\}^{m\times m}$ and a hash value $\mathbf{h}^* \in \mathbb{Z}_q^{n\times n}$ randomly, and calculates $\mathbf{A}_0 = \mathbf{A}\mathbf{R}^*_{\mathbf{ck}} - \mathbf{h}^*\mathbf{G}, \mathbf{A}_1 = \mathbf{A}\mathbf{R}^*_{t,1} - t^*_1\mathbf{G}, \ldots, \mathbf{A}_d = \mathbf{A}\mathbf{R}^*_{t,d} - t^*_d\mathbf{G}$. Finally, $\mathcal{B}$ sends $pp = \{n, m, q, \sigma, d, \mathbf{G}, \mathbf{u}, H\}$ and $pk_R = \{\mathbf{A}, \mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_d\}$ to $\mathcal{A}$, and keeps $sk_{R,\emptyset} = \mathbf{T_A}$.
- **Phase 1**: $\mathcal{B}$ responds all queries from $\mathcal{A}$ as showed in $\widehat{\textbf{Game 1}}$.
- **Challenge**: After receiving two challenge keywords $\mathbf{ck}^*_0, \mathbf{ck}^*_1 \in \mathbb{Z}_q^n$ which have not been queried in **Phase 1**, $\mathcal{B}$ selects a random bit $b \in \{0,1\}$, and calculates the ciphertext of $\mathbf{ck}^*_b$ as follows:

$$\mathbf{c}^*_1 = (\mathbf{I}_m \mid \mathbf{R}_{\mathbf{ck}^*} \mid \mathbf{R}^*_{t,1} \mid \ldots \mid \mathbf{R}^*_{t,d})^\top \mathbf{c_A}, c^*_2 = c_\mathbf{u}.$$

  Then, $\mathcal{B}$ returns the challenge ciphertext $\mathrm{CT}^*_b = (\mathbf{c}^*_1, c^*_2)$ with the challenge tag list T^* to $\mathcal{A}$.
- **Phase 2**: $\mathcal{B}$ responds all queries from $\mathcal{A}$ as showed in **Phase 1**, but either $\mathbf{ck}^*_0$ or $\mathbf{ck}^*_1$ cannot be queried in $\mathcal{O}_{\mathrm{CT}}$ and $\mathcal{O}_{\mathrm{TD}}$.
- **Guess**: $\mathcal{A}$ outputs a guess as to whether it interacts with $\widehat{\textbf{Game 1}}$ and $\widehat{\textbf{Game 2}}$. After that, $\mathcal{B}$ returns the guess from $\mathcal{A}$ to solve the $\mathrm{LWE}_{n,m,q,\chi}$ assumption.

Analysis: If the LWE instances are pseudorandom, we have:

$$\begin{aligned}
\mathbf{c}^*_1 &= (\mathbf{I}_m \mid \mathbf{R}_{\mathbf{ck}^*} \mid \mathbf{R}^*_{t,1} \mid \ldots \mid \mathbf{R}^*_{t,d})^\top (\mathbf{A}^\top\mathbf{s} + \mathbf{e}_0) \\
&= (\mathbf{A} \mid \mathbf{A}^*_{\mathbf{ck}} \mid \mathbf{A}_1 + t^*_1\mathbf{G} \mid \ldots \mid \mathbf{A}_d + t^*_d\mathbf{G})^\top \mathbf{s} \\
&\quad + (\mathbf{e}_0^\top \mid {\mathbf{e}^*_{\mathbf{ck}}}^\top \mid {\mathbf{e}^*_{t,1}}^\top \mid \ldots \mid {\mathbf{e}^*_{t,d}}^\top) \\
&= {\mathbf{H}^*}^\top \mathbf{s} + \mathbf{e}^*_1 \in \mathbb{Z}_q^{(d+2)m}, \\
c^*_2 &= c_\mathbf{u} = \mathbf{u}^\top\mathbf{s} + e_2 \in \mathbb{Z}_q,
\end{aligned}$$

where $\mathbf{H}^* = (\mathbf{A} \mid \mathbf{A}^*_{\mathbf{ck}} \mid \mathbf{A}_1 + t^*_1\mathbf{G} \mid \ldots \mid \mathbf{A}_d + t^*_d\mathbf{G})$ and $\mathbf{e}^*_1 = (\mathbf{e}_0^\top \mid {\mathbf{e}^*_{\mathbf{ck}}}^\top \mid {\mathbf{e}^*_{t,1}}^\top \mid \ldots \mid {\mathbf{e}^*_{t,d}}^\top)$. Since $\mathbf{A}\mathbf{R}^*_{\mathbf{ck}} = \mathbf{A}_0 + \mathbf{h}^*\mathbf{G} = \mathbf{A}_0 + H(\mathbf{ck}^*_b)\mathbf{G} = \mathbf{A}^*_{\mathbf{ck}}$ holds if and only if $H(\mathbf{ck}^*_b) = \mathbf{h}^*$, the distribution of CT^*_b can correspond to $\widehat{\textbf{Game 1}}$ with probability $\frac{1}{q_H}$.

If the LWE instances are random, we have $\mathbf{c}^*_1$ and c^*_2 are uniform over $\mathbb{Z}^{(d+2)m}$ and $\mathbb{Z}_q$, respectively according to Lemma 7. Thus, the distribution of CT^*_b corresponds to $\widehat{\textbf{Game 2}}$. Consequently, if the advantage of $\mathcal{A}$ to distinguish between $\widehat{\textbf{Game 1}}$ and $\widehat{\textbf{Game 2}}$ is a non-negligible value ϵ, the advantage of the algorithm $\mathcal{B}$ to solve $\mathrm{LWE}_{n,m,q,\chi}$ assumption is $\frac{\epsilon}{q_H}$, which is also non-negligible. $\square$

7 Performance Evaluation and Comparison

We evaluate the computational and communication overhead of PunSearch. To ensure fairness, we compare our results with other lattice-based schemes [26], [30], [31], [22], [33], [34]. All experiments are implemented in Python on a Mac OS system with an Apple M2 CPU, 8GB RAM, and 256GB SSD. Each round of the experiment is carried out independently.

7.1 Computational Overhead Analysis

To begin with, we perform a theoretical analysis of the computational overhead of **Encrypt**, **Trapdoor**, and **Search** algorithms in our PunSearch scheme together with other lattice-based SE schemes (FS-PEKS [26], ABAEKS [30], Re-PAEKS [31], and IBEDKS [22]), as summarized in Table 3.

Concretely, considering the **Encrypt** algorithm, T_H denotes the time cost of $H(\mathbf{ck})$, n^2mT_{Mul} indicates the time cost of $\mathbf{A}_{\mathbf{ck}}$, $(d+2)m^2T_{Mul}$ corresponds to the time cost of $\mathbf{e}_1$, $dnmT_{Mul}$ reflects the time cost of $\mathbf{A}_{\mathbf{ck},T}$, $(d+2)nmT_{Mul}$ and nT_{Mul} represent the time cost of $\mathbf{c}_1$, and c_2, respectively. Therefore, the total cost for encrypting a keyword in PunSearch is $T_H+[n^2m+(d+2)m^2+(2d+2)nm+n]T_{Mul}$. It indicates that PunSearch is more efficient than others as it only requires hash and matrix multiplication operations, without the time-consuming matrix inversion and lattice basis sampling operations. Due to $T_{NBD} >> T_{SP}+T_{SL} > T_{GSP}$, the computational overhead of our **Trapdoor** algorithm is considerably lower than FS-PEKS and IBEDKS, and similar to ABAEKS and Re-PAEKS. Note that T_{Ect} is essentially a constant-level multiplication operation. With regard to the **Search** algorithm, the number of multiplications in these five schemes is proportional to m. As we additionally provide the puncture property, our **Search** algorithm relies on the number of punctured tags ψ. Not only the theoretical analysis, but also the results of simulation experiments validate the same, as depicted in Fig. 8(c).

Furthermore, we conduct the experimental simulation analysis as follows. For fairness and security, we configure the parameters $q = 4097$, $n = 16$, $m = \lceil 2n \log q \rceil = 385$, $|att| = 10$, $N = 10$, $d = 10$ and $\psi = 1$, for simulation of our PunSearch scheme and other prior arts.

On the one hand, we compare the computational overhead of the **Encrypt**, **Trapdoor**, and **Search** algorithms in PunSearch with others (FS-PEKS [26], ABAEKS [30], Re-PAEKS [31], and IBEDKS [22]). Table 4 illustrates these results for the number of keywords $k = 1$. Specifically, the time costs of these three algorithms in PunSearch are 18.79ms, 44.35ms, and 0.01ms, which are just $0.064\times$, $0.005\times$ and $0.050\times$ compared to [26], [30], [31], [22], respectively for the best cases. Fig. 8 depicts the overhead of **Encrypt**, **Trapdoor** and **Search** algorithms in these schemes, with k ranging from 1 to 100. It observes that the computational overheads of the three algorithms in PunSearch are proportional to k, with the **Encrypt** and **Search** algorithms showing a more gradual increase compared to prior arts. Due to the larger matrix size inputted to GenSamplePre algorithm, the time cost to generate a trapdoor in PunSearch is only higher than one scheme [31]. As trapdoor generation is a one-time procedure, this result is acceptable in practice, which is a trade-off between functionality and practicality.

On the other hand, we also provide the computational overheads of the **Puncture** algorithm in our design together with other PE schemes over lattice (PIBE [33] and PHIBE [34]), as depicted in Fig. 9, for comparing the puncture property. Specifically, Fig. 9(a) illustrates the time cost with respect to d when $\psi = 2$, while Fig. 9(b) presents the time cost in relation to ψ when $d = 10$. In our design, the matrix sizes used in the ExtendRight and RandBasis algorithms are smaller compared to those in PIBE and PHIBE. This difference makes the PunSearch scheme markedly more efficient than the other in the context of **Puncture** algorithm. For instance, the time costs for this algorithm in PIBE, PHIBE, and PunSearch are 96.77ms, 173.23ms, and 53.80ms, respectively, when $d = 10$ and $\psi = 2$. As ψ increases, the time cost of ExtendRight and RandBasis algorithms rises dramatically, Consequently, the efficiency of the **Puncture** algorithm becomes more drastically impacted by the number of punctured tags ψ.

7.2 Communication Overhead Analysis

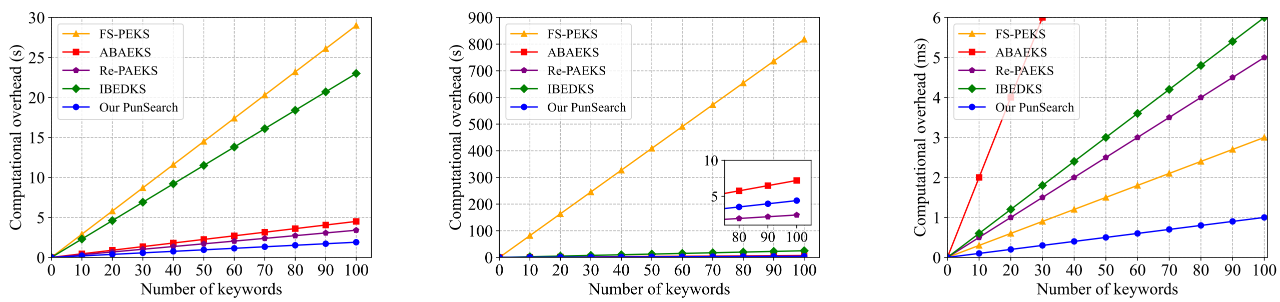
In Table 5, we compare the communication overhead of PunSearch with other lattice-based SE schemes [26], [30], [31], [22]. In the **KeyGen**$_R$ algorithm, the secret key size in PunSearch is close to [26], [31], and [22], and smaller than that in [30]. Since the secret key must be transmitted to a data receiver after executing the **KeyGen**$_R$ algorithm, a smaller secret key size is more convenient for storage in practical scenarios. For the **Encrypt** algorithm, the ciphertext size in PunSearch relies on d. The ciphertext sizes are related to parameters such as l and $|att|$ in FS-PEKS and ABAEKS, while Re-PAEKS and IBEDKS have relatively constant costs. Although our design does not outperform the other schemes in terms of ciphertext size, it additionally offers fine-grained searchability revocation, which is an acceptable trade-off. Compared to ciphertext, PunSearch incurs a lower communication overhead for transmitting a trapdoor, and its size increases linearly as the number of punctured tags ψ grows.

We set the parameters $|\mathbb{Z}_q| = \lceil \log q \rceil = 13$, $l = 10$, $|att| = 10$, $N = 10$, $d = 10$, $\psi = 1$, and give an experimental comparison of communication overhead for **KeyGen**$_R$, **Encrypt** and **Trapdoor** algorithms in PunSearch and others ([26], [30], [31], [22]), as depicted in Fig. 10. In particular, in Fig. 10(a), we evaluate the key size of different schemes as the communication overhead of the **KeyGen**$_R$ algorithm, where our design is more efficient than other schemes. In Fig. 10(b), the ciphertext sizes of the five schemes are 6.13KB, 8.55KB, 4.89KB, 1.22KB and 7.33KB, respectively. Since our scheme adds tags to the data ciphertext, the ciphertext size of PunSearch does not have an advantage over the other schemes, but this is acceptable. Fig. 10(c) displays the communication overhead of the **Trapdoor** algorithm in PunSearch and these four schemes. The trapdoor size of PunSearch is only 1.83KB, significantly lower than those of ABAEKS and Re-PAEKS, and PunSearch also possesses the searchability revocation property for specific keywords compared to FS-PEKS and IBEDKS. Our PunSearch scheme can effectively reduce the network transmission burden during the communication between the data receiver and the MCS. Fig. 10 provides strong support for the theoretical results presented in Table 5. Kindly note that the ciphertext

TABLE 3
Theoretical computational overhead comparison

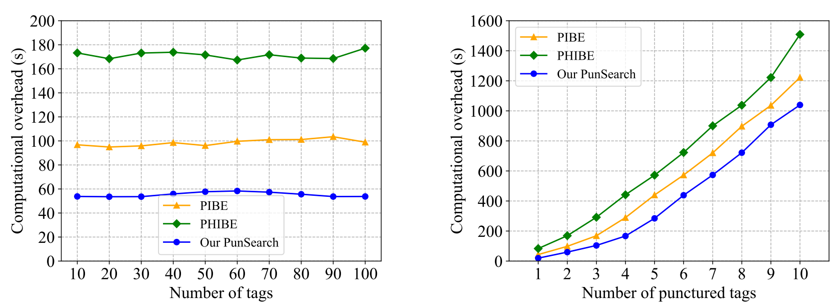
Schemes	Encrypt	Trapdoor	Search
FS-PEKS [26]	$T_H + T_{Inv} + (nm^2 + nml + nl)T_{Mul}$	$T_H + T_{Inv} + T_{NBD} + T_{SP} + nm^2 T_{Mul}$	mlT_{Mul}
ABAEKS [30]	$T_H + T_{SL} + [(2\|att\| + 3)nm + n^2 + (\|att\| + 1)m^2]T_{Mul}$	$T_H + T_{Epk} + T_{SL} + (n^2 + m^2 + 3nm)T_{Mul}$	$T_{Ect} + 5mT_{Mul}$
Re-PAEKS [31]	$T_H + T_{SL} + (n^2 + 3m^2 + 5nm)T_{Mul}$	$T_H + T_{SL} + (n^2 + 3m^2 + 5nm)T_{Mul}$	$8mT_{Mul}$
IBEDKS [22]	$2T_H + T_{Inv} + [n^2 m + nm^2 + (n + m)^2 + n + N - 1]T_{Mul}$	$(l + 1)T_H + T_{Inv} + T_{SP} + T_{SL} + (nm^2 + N^2 + N - 2)T_{Mul}$	$(2m + 1)T_{Mul}$
Our PunSearch	$T_H + [n^2 m + (d + 2)m^2 + (2d + 2)nm + n]T_{Mul}$	$T_H + \psi T_{Epk} + T_{GSP} + nm^2 T_{Mul}$	$\psi T_{Ect} + (\psi + 2)mT_{Mul}$

Note: l: The security-level of testing; $|att|$: The length of attributes; N: The maximum amount of keyword defined in IBEDKS [22]; d: The number of tags; ψ: The number of punctured tags; k: The number of keywords; T_H: The time cost of hash function; T_{Inv}: The time cost of matrix inversion; T_{Mul}: The time cost of multiplication; T_{NBD}: The time cost of NewBasisDel algorithm; T_{SP}: The time cost of SamplePre algorithm; T_{SL}: The time cost of SampleLeft algorithm; T_{Epk}: The time cost of Eval_{pk} algorithm; T_{Ect}: The time cost of Eval_{ct} algorithm; T_{GSP}: The time cost of GenSamplePre algorithm.



(a) **Encrypt** algorithm (b) **Trapdoor** algorithm (c) **Search** algorithm

Fig. 8. Computational overhead comparison between our PunSearch scheme and other PEKS schemes [26], [30], [31], [22] with the number of keywords k.



(a) **Puncture** algorithm (w.r.t. d) (b) **Puncture** algorithm (w.r.t. ψ)

Fig. 9. Computational overhead evaluation of the **Puncture** algorithm in our PunSearch scheme and other PE schemes [33], [34].

TABLE 4
Computational overhead comparison in $k = 1$ (ms)

Schemes	Encrypt	Trapdoor	Search
FS-PEKS [26]	292.11	8181.90	0.03
ABAEKS [30]	45.37	72.41	0.20
Re-PAEKS [31]	33.80	24.06	0.05
IBEDKS [22]	229.40	245.98	0.06
Our PunSearch	18.79	44.35	0.01

TABLE 5
Theoretical communication overhead comparison

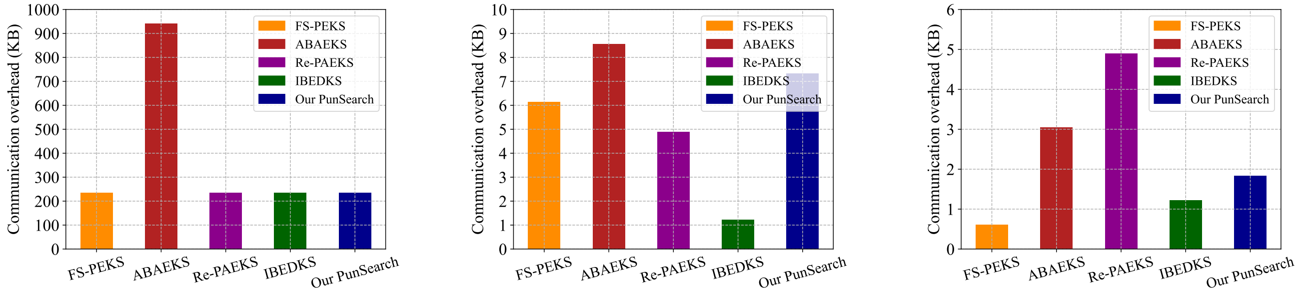
Schemes	Secret key	Ciphertext	Trapdoor
FS-PEKS [26]	$m^2\|\mathbb{Z}_q\|$	$l(m+1)\|\mathbb{Z}_q\|$	$m\|\mathbb{Z}_q\|$
ABAEKS [30]	$4m^2\|\mathbb{Z}_q\|$	$(\|att\|+4)m\|\mathbb{Z}_q\|$	$5m\|\mathbb{Z}_q\|$
Re-PAEKS [31]	$m^2\|\mathbb{Z}_q\|$	$8m\|\mathbb{Z}_q\|$	$8m\|\mathbb{Z}_q\|$
IBEDKS [22]	$m^2\|\mathbb{Z}_q\|$	$(2m+1)\|\mathbb{Z}_q\|$	$(2m+1)\mathbb{Z}_q$
Our PunSearch	$m^2\|\mathbb{Z}_q\|$	$[(d+2)m+1]\|\mathbb{Z}_q\|$	$(\psi+2)m\|\mathbb{Z}_q\|$

Note: $|\mathbb{Z}_q|$: The size of an element in $\mathbb{Z}_q$.

and trapdoor size of PunSearch are $[(d+2)m+1]|\mathbb{Z}_q|$ and $(\psi+2)m|\mathbb{Z}_q|$, respectively, both of which are proportional to d and ψ. Fig. 11 displays the details of our evaluation.

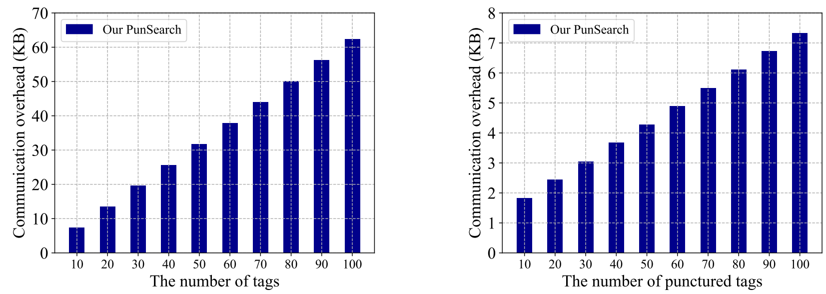
8 Conclusion

In this paper, we have presented the first puncturable encrypted search scheme over lattice for outsourced data



(a) Secret key size (b) Ciphertext size (c) Trapdoor size

Fig. 10. Communication overhead comparison between our PunSearch scheme and other PEKS schemes [26], [30], [31], [22].



(a) Ciphertext size (b) Trapdoor size

Fig. 11. Communication overhead evaluation of the **Puncture** algorithm in PunSearch scheme with the number of tags d and punctured tags ψ.

privacy-preserving in mobile cloud, named PunSearch. Inspired by the PE primitive, we have punctured the secret key of data receivers through the Puncture algorithm. After that, we have designed a novel trapdoor generation and search algorithm to match a ciphertext with a trapdoor with searchability. Our scheme provides fine-grained searchability revocation for specific keywords and resists quantum computing attacks. The rigorous security analysis has revealed that PunSearch enjoys IND-Pun-CKA security in the ROM. Comprehensive performance results have demonstrated that PunSearch is more efficient than other prior art in the context of computational overheads. As future work, we will introduce an authenticated encryption to PunSearch to defend against insider keyword guessing attacks.

References

[1] Y. Miao, F. Li, X. Li, Z. Liu, J. Ning, H. Li, K.-K. R. Choo, and R. H. Deng, "Time-controllable keyword search scheme with efficient revocation in mobile e-health cloud," *IEEE Transactions on Mobile Computing*, vol. 23, no. 5, pp. 3650–3665, 2023.

[2] M. Wang, J. Yu, W. Shen, and R. Hao, "Privacy-preserving time-based auditing for secure cloud storage," *IEEE Transactions on Information Forensics and Security*, vol. 19, pp. 7866–7878, 2024.

[3] S. Xu, X. Chen, Y. Guo, S.-M. Yiu, S. Gao, and B. Xiao, "Efficient and secure post-quantum certificateless signcryption with linkability for iomt," *IEEE Transactions on Information Forensics and Security*, vol. 20, pp. 1119–1134, 2025.

[4] Y. Miao, G. Wang, X. Li, H. Li, K.-K. R. Choo, and R. H. Deng, "Efficient and secure geometric range search over encrypted spatial data in mobile cloud," *IEEE Transactions on Mobile Computing*, vol. 24, no. 3, pp. 1621–1635, 2025.

[5] G. Chi, Q. Huang, and C. Shi, "Privacy-preserving and autonomous-path access delegation for mobile cloud healthcare systems," *IEEE Transactions on Mobile Computing*, no. 01, pp. 1–14, 2025.

[6] X. Qu, S. Wang, K. Li, J. Huang, and X. Cheng, "Tidyblock: A novel consensus mechanism for dag-based blockchain in iot," *IEEE Transactions on Mobile Computing*, vol. 24, no. 2, pp. 722–735, 2025.

[7] G. Xu, L. Lei, Y. Mao, Z. Li, X.-B. Chen, and K. Zhang, "Cbrfl: A framework for committee-based byzantine-resilient federated learning," *Journal of Network and Computer Applications*, vol. 238, p. 104165, 2025.

[8] G. Xu, D.-L. Kong, K. Zhangs, S. Xu, Y. Cao, Y. Mao, J. Duan, J. Kang, and X.-B. Chen, "A model value transfer incentive mechanism for federated learning with smart contracts in aiot," *IEEE Internet of Things Journal*, vol. 12, no. 3, pp. 2530–2544, 2025.

[9] H.-J. Chien, H. Khalili, A. Hass, and N. Sehatbakhsh, "Enc2: Privacy-preserving inference for tiny iots via encoding and encryption," in *Proceedings of the 29th Annual International Conference on Mobile Computing and Networking*, 2023, pp. 1–16.

[10] K. Routray and P. Bera, "Efficient and secure cloud data sharing using cp-abe supporting dynamic attributes," in *Proceedings of the 30th Annual International Conference on Mobile Computing and Networking*, 2024, pp. 2245–2247.

[11] C. Liu, M. Xu, H. Guo, X. Cheng, Y. Xiao, D. Yu, B. Gong, A. Yerukhimovich, S. Wang, and W. Lyu, "Tbac: A tokoin-based accountable access control scheme for the internet of things," *IEEE Transactions on Mobile Computing*, vol. 23, no. 5, pp. 6133–6148, 2024.

[12] M. Wang, Y. Miao, Y. Guo, H. Huang, C. Wang, and X. Jia, "Aesm 2 attribute-based encrypted search for multi-owner and multi-user distributed systems," *IEEE Transactions on Parallel and Distributed Systems*, vol. 34, no. 1, pp. 92–107, 2022.

[13] Y. Xu, H. Cheng, X. Liu, C. Jiang, X. Zhang, and M. Wang, "Pcse: Privacy-preserving collaborative searchable encryption for group data sharing in cloud computing," *IEEE Transactions on Mobile Computing*, vol. 24, no. 5, pp. 4558–4572, 2025.

[14] Y. Wang, X. Li, Y. Miao, Q. Tong, X. Liu, and R. H. Deng, "Plrq: Practical and less leakage range query over encrypted mobile

cloud data," *IEEE Transactions on Mobile Computing*, vol. 24, no. 5, pp. 4183–4201, 2025.

[15] K. Routray, S. Shukla, and P. Bera, "Multi-client searchable encryption with granular access control for cloud-assisted iot," in *Proceedings of the 30th Annual International Conference on Mobile Computing and Networking*, 2024, pp. 1620–1622.

[16] D. Zhang, S. Wang, Q. Zhang, and Y. Zhang, "Attribute based conjunctive keywords search with verifiability and fair payment using blockchain," *IEEE Transactions on Services Computing*, vol. 16, no. 6, pp. 4168–4182, 2023.

[17] L. Chen, S. Xu, C. Jin, H. Zhang, and J. Weng, "Partially hidden policy attribute-based multi-keyword searchable encryption with verification and revocation," *IEEE Transactions on Mobile Computing*, pp. 1–16, 2025.

[18] W. Li, "Multi-receiver data authorization with data search for data sharing in cloud-assisted iov," *IEEE Transactions on Intelligent Transportation Systems*, vol. 25, no. 5, pp. 4233–4250, 2024.

[19] L. Chen, J. Li, J. Li, and J. Weng, "Paess: Public-key authentication encryption with similar data search for pay-per-query," *IEEE Transactions on Information Forensics and Security*, vol. 19, pp. 9910–9923, 2024.

[20] L. Cheng and F. Meng, "Server-aided public key authenticated searchable encryption with constant ciphertext and constant trapdoor," *IEEE Transactions on Information Forensics and Security*, vol. 19, pp. 1388–1400, 2023.

[21] X. Zhang, C. Huang, D. Gu, J. Zhang, and H. Wang, "Bib-mks: post-quantum secure biometric identity-based multi-keyword search over encrypted data in cloud storage systems," *IEEE Transactions on Services Computing*, vol. 16, no. 1, pp. 122–133, 2021.

[22] Z. Lin, H. Li, X. Chen, M. Xiao, and Q. Huang, "Identity-based encryption with disjunctive, conjunctive and range keyword search from lattices," *IEEE Transactions on Information Forensics and Security*, vol. 19, pp. 8644–8657, 2024.

[23] S. Xu, X. Chen, Y. Guo, Y. Yang, S. Wang, S.-M. Yiu, and X. Cheng, "Lattice-based forward secure multi-user authenticated searchable encryption for cloud storage systems," *IEEE Transactions on Computers*, vol. 74, no. 5, pp. 1663–1677, 2025.

[24] W. Sun, S. Yu, W. Lou, Y. T. Hou, and H. Li, "Protecting your right: Verifiable attribute-based keyword search with fine-grained owner-enforced search authorization in the cloud," *IEEE Transactions on Parallel and Distributed Systems*, vol. 27, no. 4, pp. 1187–1198, 2014.

[25] W. Zhang, Y. Lin, S. Xiao, J. Wu, and S. Zhou, "Privacy preserving ranked multi-keyword search for multiple data owners in cloud computing," *IEEE Transactions on Computers*, vol. 65, no. 5, pp. 1566–1577, 2015.

[26] X. Zhang, C. Xu, H. Wang, Y. Zhang, and S. Wang, "Fs-peks: Lattice-based forward secure public-key encryption with keyword search for cloud-assisted industrial internet of things," *IEEE Transactions on dependable and secure computing*, vol. 18, no. 3, pp. 1019–1032, 2019.

[27] Y. Lu and J. Li, "Privacy-preserving and forward public key encryption with field-free multi-keyword search for cloud encrypted data," *IEEE Transactions on Cloud Computing*, vol. 11, no. 4, pp. 3619–3630, 2023.

[28] S. Xu, Y. Cao, X. Chen, Y. Zhao, and S.-M. Yiu, "Post-quantum public-key authenticated searchable encryption with forward security: General construction, and applications," in *International Conference on Information Security and Cryptology*. Springer, 2023, pp. 274–298.

[29] Y. Cao, S. Xu, X. Chen, Y. He, and S. Jiang, "A forward-secure and efficient authentication protocol through lattice-based group signature in vanets scenarios," *Computer Networks*, vol. 214, p. 109149, 2022.

[30] F. Luo, H. Wang, C. Lin, and X. Yan, "Abaeks: Attribute-based authenticated encryption with keyword search over outsourced encrypted data," *IEEE Transactions on Information Forensics and Security*, vol. 18, pp. 4970–4983, 2023.

[31] F. Luo, H. Wang, and X. Yan, "Re-paeks: Public-key authenticated re-encryption with keyword search," *IEEE Transactions on Mobile Computing*, vol. 23, no. 10, pp. 10 077–10 092, 2024.

[32] G. Xu, S. Xu, Y. Cao, K. Xiao, Y. Mao, X.-B. Chen, M. Dong, and S. Yu, "Aaq-peks: An attribute-based anti-quantum public key encryption scheme with keyword search for e-healthcare scenarios," *Peer-to-Peer Networking and Applications*, vol. 18, no. 2, pp. 1–21, 2025.

[33] P. Dutta, W. Susilo, D. H. Duong, and P. S. Roy, "Puncturable identity-based encryption from lattices," in *Information Security and Privacy: 26th Australasian Conference (ACISP)*. Springer, 2021, pp. 571–589.

[34] P. Dutta, M. Jiang, D. H. Duong, W. Susilo, K. Fukushima, and S. Kiyomoto, "Hierarchical identity-based puncturable encryption from lattices with application to forward security," in *Proceedings of the 2022 ACM on Asia Conference on Computer and Communications Security*, 2022, pp. 408–422.

[35] D. Boneh, G. Di Crescenzo, R. Ostrovsky, and G. Persiano, "Public key encryption with keyword search," in *Advances in Cryptology-EUROCRYPT*. Springer, 2004, pp. 506–522.

[36] L. Xu, W. Li, F. Zhang, R. Cheng, and S. Tang, "Authorized keyword searches on public key encrypted data with time controlled keyword privacy," *IEEE Transactions on Information Forensics and Security*, vol. 15, pp. 2096–2109, 2019.

[37] P. Xu, S. Tang, P. Xu, Q. Wu, H. Hu, and W. Susilo, "Practical multi-keyword and boolean search over encrypted e-mail in cloud server," *IEEE Transactions on Services Computing*, vol. 14, no. 6, pp. 1877–1889, 2019.

[38] K. Zhang, Z. Jiang, J. Ning, and X. Huang, "Subversion-resistant and consistent attribute-based keyword search for secure cloud storage," *IEEE Transactions on Information Forensics and Security*, vol. 17, pp. 1771–1784, 2022.

[39] M. D. Green and I. Miers, "Forward secure asynchronous messaging from puncturable encryption," in *2015 IEEE Symposium on Security and Privacy*. IEEE, 2015, pp. 305–320.

[40] T. V. X. Phuong, R. Ning, C. Xin, and H. Wu, "Puncturable attribute-based encryption for secure data delivery in internet of things," in *IEEE INFOCOM 2018-IEEE conference on computer communications*. IEEE, 2018, pp. 1511–1519.

[41] D. Ghopur, J. Ma, X. Ma, J. Hao, T. Jiang, and X. Wang, "Puncturable key-policy attribute-based encryption scheme for efficient user revocation," *IEEE Transactions on Services Computing*, vol. 16, no. 6, pp. 3999–4011, 2023.

[42] H. Cui and X. Yi, "Secure internet of things in cloud computing via puncturable attribute-based encryption with user revocation," *IEEE Internet of Things Journal*, vol. 11, no. 2, pp. 3662–3670, 2024.

[43] W. Susilo, D. H. Duong, H. Q. Le, and J. Pieprzyk, "Puncturable encryption: a generic construction from delegatable fully key-homomorphic encryption," in *European Symposium on Research in Computer Security (ESORICS)*. Springer, 2020, pp. 107–127.

[44] M. Yang, H. Wang, and D. He, "Puncturable attribute-based encryption from lattices for classified document sharing," *IEEE Transactions on Information Forensics and Security*, vol. 19, pp. 4028–4042, 2024.

[45] M. Yang, H. Wang, and D. He, "Pm-abe: Puncturable bilateral fine-grained access control from lattices for secret sharing," *IEEE Transactions on Dependable and Secure Computing*, vol. 22, no. 2, pp. 1210–1223, 2025.

[46] S. Xu, Y. Cao, X. Chen, Y. Guo, Y. Yang, F. Guo, and S.-M. Yiu, "Post-quantum searchable encryption supporting user-authorization for outsourced data management," in *Proceedings of the 33rd ACM International Conference on Information and Knowledge Management*, 2024, pp. 2702–2711.

[47] O. Regev, "On lattices, learning with errors, random linear codes, and cryptography," *Journal of the ACM (JACM)*, vol. 56, no. 6, pp. 1–40, 2009.

[48] D. Boneh, C. Gentry, S. Gorbunov, S. Halevi, V. Nikolaenko, G. Segev, V. Vaikuntanathan, and D. Vinayagamurthy, "Fully key-homomorphic encryption, arithmetic circuit abe and compact garbled circuits," in *Advances in Cryptology–EUROCRYPT*. Springer, 2014, pp. 533–556.

[49] D. Micciancio and C. Peikert, "Trapdoors for lattices: Simpler, tighter, faster, smaller," in *Annual International Conference on the Theory and Applications of Cryptographic Techniques*. Springer, 2012, pp. 700–718.

[50] D. Cash, D. Hofheinz, E. Kiltz, and C. Peikert, "Bonsai trees, or how to delegate a lattice basis," *Journal of cryptology*, vol. 25, pp. 601–639, 2012.

[51] S. Agrawal, D. Boneh, and X. Boyen, "Efficient lattice (h) ibe in the standard model," in *Advances in Cryptology–EUROCRYPT*. Springer, 2010, pp. 553–572.